Growth Risk of Nontraded Industries and Asset Pricing *

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Abstract

This paper shows that output fluctuations in nontraded industries are a central risk factor driving asset prices in all countries. Because nontraded outputs are consumed domestically, their growth risk is mostly non-diversifiable. In interest rate markets, fluctuations in the growth of industries with higher nontradability feed greater risk to the economy and lower interest rates. In currency markets, these fluctuations induce co-movements between countries' marginal utilities and exchange rates, and generate large currency premia. The nontraded output growth risk presents a rationale for why known funding and investment currencies are associated with economies of diverse sizes.

JEL-Classification: F3, F31, F4, G0, G12, G15.

Keywords: Nontraded output risk, Exchange rate, Currency carry trade, Interest rate.

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1 Introduction

The rational theory and practice of asset pricing center around fundamental principles of the risk return tradeoff, diversification, and no arbitrage. Movements in an economy's nontraded-sector output should play a key role in the determination of domestic asset prices and their differentials across economies, because these risks are not easily diversified even in an arbitrage-free and complete international market. This paper shows that the nontraded output growth risk is indeed an important determinant of international asset prices. We adopt a consumption-based equilibrium economic setting, with multiple countries, nontraded and traded goods, and trade costs. Our model features and accounts for *partially traded* goods and services as they actually are in reality. This property allows us to estimate the effects of nontraded output risk that are robust to possible classification errors in macro data. Using data from the Organisation for Economic Co-operation and Development (OECD) economies, we verify new implications of nontraded output growth risk for the interest rate and currency markets.

The main insight of this paper is that the nontradability of a country-specific output amplifies the impact of the output's risk on the host country of any size. From this insight follow all our key conceptual results, which are empirically verified in the paper. First, at the country level, the fluctuations in gross domestic product (GDP) growth of less open-to-trade economies pose greater risk, induce higher precautionary savings motives, and lower home interest rates relative to a cross section of economies. Second, at the industry level, the fluctuations in the output growth of less traded industries also place stronger downward pressure on interest rates. Third, the nontraded output growth risk regulates consumption allocation, moves investors' marginal utilities and exchange rates simultaneously, breaks the uncovered interest rate parity, and generates currency premia. In currency markets, the carry trade strategies that expose investors to larger nontraded output growth risk offer higher returns on average. Fourth, the nontraded output risk matters for the host country's asset pricing both on its own and on its coupling with the country's size. In contrast, country-specific *traded* output growth risks are less prominent, because they are subject to diversification via international trades.

The nontraded sector produces goods and services that cannot be consumed outside of the host country. It includes some important sectors to the economy, such as wholesale and retail trade, hotels and restaurants, real estate, financial intermediaries, transportation, information and communications, art and entertainment, and domestic business activities. Three key features of nontraded outputs stand out. First, nontraded outputs feed the lion's share to the GDP and national aggregate consumption in all countries. Figure 1 shows that the ratio of real nontraded output over GDP is substantial across the OECD economies, ranging from 0.45 to 0.8, and almost 0.7 for the U.S. Second, the *tradabilities*, commonly measured as (one half of) the ratio of total import plus export over output, of key nontraded industries are indeed low. Table 1 shows that the tradabilities of various different nontraded sectors are mostly in the order of few percentage points, and rarely exceed 15% across a host of countries. Third, the magnitude of nontraded output growth risk is largely unrelated to the size of host countries. Figure 2 shows a heterogeneous pattern between the relative size (measured as a country's real GDP over OECD's total real GDP) and nontraded output growth volatility across countries, with the U.S. having a large size and a small nontraded output volatility in one extreme, and Switzerland having a small size and a large nontraded output volatility in the other. These stylized facts imply that the nontraded output growth volatilities would pose a major source of risk to economies of all sizes, and should be reflected in the interest rates, exchange rates and currency returns. We discuss these three asset pricing quantities in turn.

First, the primary force underlying the impact of nontraded growth risk on interest rates is the precautionary savings motives of market participants. Under these motives, volatilities of the home nontraded output, trade partners' nontraded outputs, and the global (aggregate) traded output, all act to depress the home interest rate because these volatilities induce uncertainties in the home consumptions. However, as mentioned above, while the country-specific traded output risk is largely internationalized and diversified in the global pool of traded goods, the nontraded output risk is primarily internalized. Consequently, the home nontraded output volatility impacts the home interest rate directly and more strongly than does the home-specific traded output volatility. To the extent that a country is able to mitigate the nontraded output risk by substituting and adjusting the traded consumption, this adjustment is hindered by the country's size (larger countries face larger and more challenging traded consumption adjustments). Hence, the risk mitigation is partial, and presents a second and indirect impact of the nontraded output volatility coupled with the country's size on the interest rate.

Second, the primary force underlying the impact of nontraded growth risk on the exchange rate is the transmission of shocks across countries. Beyond their dominant impacts at home, nontraded output fluctuations are an important source of risk because they also matter for *all* trade partners of the home country. The transmission of home nontraded output shocks starts with these shocks being substituted and offset imperfectly by adjustments in the home traded consumption. These adjustments are facilitated by international trades, and induce fluctuations in the traded consumptions of home's trade partners by market clearing in traded goods. As the exchange rate characterizes the ratio of two countries' price numeraires, i.e., the difference of two country-specific pricing kernels (SDFs), nontraded output shocks induce fluctuations in the exchange rate. Furthermore, a country's nontraded output risk impacts exchange rate both on its own (via its direct impact on the home SDF) and on its coupling with the home size (via its indirect impact on international trade and foreign SDF).

Third, the nontraded output risks is an important factor behind carry trade profits because it induces simultaneous movements in (hence, the correlation between) SDFs and the exchange rate. Why do certain currency pairs tend to generate profits, whereas others incur losses in the currency market? Let us consider a strategy of borrowing the home currency and lending the foreign currency. An adverse shock to the foreign nontraded output simultaneously causes the foreign currency to appreciate (i.e., a surge in the foreign consumption basket value due to the scarcity of foreign nontraded goods) and the home traded consumption to drop (i.e., a surge in the foreign traded consumption to substitute for the foreign nontraded output loss). That is, with respect to the foreign nontraded risk, this strategy pays well when home investors highly value the consumption, and vice versa. From the perspective of home investors, this carry trade is a hedge against foreign nontraded output shocks, and thus commands a low, possibly negative, expected excess return to home investors with respect to this risk. Note that the adverse foreign nontraded shock induces the foreign currency's appreciation on its own, and the home traded consumption's drop via its coupling with the foreign country's size. Therefore, this carry trade expected return is lower when the foreign nontraded sector is riskier or its economy is larger. By the same argument, this carry trade is not a hedge against the home nontraded output growth risk, and thus commands a high expected return with respect to this risk. The overall expected profit (or loss) of the carry trade is determined by whether the home (or foreign) nontraded output growth risk, mediated by their respective sizes, dominates. Specifically, when home nontraded output sector is sufficiently more volatile and larger than that of the foreign counterpart, short the home and long the foreign currency tend to generate positive expected returns to compensate home investors for bearing the dominating home nontraded risk in the carry trades, and vice versa.

Conceptually, it is important to differentiate the roles of nontraded output growth risks and economy's size in international asset pricing. The nontraded output growth risk enters the setting both as a coupled factor with economy's size and as a stand-alone factor. A country's nontraded output growth risk couples with its size when this risk propagates and affects other countries via consumption substitution, adjustments, and international trades. In this process, the home nontraded output growth risk is *amplified* by the home size in its impact on foreign traded consumptions and foreign SDFs, but is *subdued* by the home size in its impact on the home traded consumption and home SDF. Apart from this coupling, a country's nontraded output growth volatility on its own is a stand-alone risk factor to the host country because the nontraded output is consumed entirely at home. The overall impact of the nontraded output growth risk combines contributions from these two factors. Empirically, while nontraded output risks do not align with sizes across economies (Figure 2), their coupling exhibits a remarkable pattern. Figure 3 shows that known "funding" currencies (Swiss Franc CHF, U.S. Dollar USD, Euro EUR, and Japanese Yen JPY) tend to occupy the large end of the spectrum of the nontraded output risk and size product. Whereas, "investment" currencies (among them, Australian Dollar AUD, and New Zealand Dollar NZD) tend to occupy the small end of the spectrum.[1] This pattern is consistent with our carry trade rationale above that long positions in these funding currencies are hedges (against funding currencies' large coupling of nontraded output risks and sizes) and deliver low expected returns.[2] Swiss Franc CHF, being associated of a small economy, would not fit the above funding currency narrative if being based solely on the economy's size. CHF fits this narrative thanks to Switzerland's significant nontraded output risk (Figure 2).

We now discuss practical aspects of tradability of goods and describe empirical designs and results of the paper. Tests concerning nontraded and traded output growth risk face empirical challenges in practice. First, as no goods are either perfectly traded or perfectly nontraded, their output data are necessarily subject to classification and measurement errors. Second, frictions in trade may hamper the diversification in the traded commodity markets. Third, the relationship between output nontradability and asset prices are of equilibrium nature. Conceptually, the adoption of a variable "iceberg transport cost" in an equilibrium setting can address these issues.[3] This is because the variation in transport cost induces simultaneous changes in trade volumes and prices, and thus fosters an equilibrium relationship between goods tradabilities and asset returns. Em-

[1] Quantitatively, the coupling that enters the currency premium is between the variance of nontraded output growth and economy's size (Proposition 2).

[2] That is, on the flip side, profitable carry trade strategies take short positions in (i.e., borrow) funding currencies, are exposed to large nontraded output risks, and deliver high expected returns.

[3] In the literature, iceberg transport cost refers to a trade friction due to which the amount of goods loss in the transition is proportional to amount of goods initially shipped.

pirically, we employ several variants of the relative magnitude between trade volumes (import and export) and outputs of a good to measure its tradability and effects in tests. The first test concerns interest rates and output growth risk at the industry level. We regress real interest rates on output growth volatilities of various industries, their tradabilities, and the interaction term, while controlling for other variables. Table 5 shows that across OECD economies and on average, one standard deviation in output growth depresses the real interest rate by an additional 5% when the output's classification changes from perfectly traded to perfectly nontraded. Another test shows a similar result; the volatility of GDP has greater effect on home interest rates when the home economy is less open to trades (i.e., having lower ratio of national trade over GDP).

The next tests concern currency trade strategies, which sort currencies according to some characteristics to verify whether these characteristics are priced in currency markets. We first perform an one-dimensional sort of OECD currencies into four portfolios based on one of the following three country-specific characteristics, namely, (i) variance of nontraded output growth, (ii) size (or GDP/total OECD GDP), and (iii) their product. We then form four carry trade strategies, each borrows USD and lends equally in currencies of the corresponding portfolio. Figure 6 exhibits monotonic carry trade mean returns for portfolios sorted on the nontraded output risk coupled with size, indicating the pricing significance of this coupling factor in currency markets. The long-short strategy on the top and bottom portfolios delivers a mean return of 3.23% and a Sharpe ratio of approximately 45%. For robustness, we then perform a two-dimensional sort of OECD currencies into 2-by-2 portfolios based on both variance of nontraded output growth and size. Figure 7 shows that, controlling for (small or large) size, lending currencies of economies of higher nontraded output risk offers lower mean returns. This pattern is consistent with the above rationale that these carry trades are hedges to the nontraded output risk of countries, whose currencies are being long in the strategy.

Our analysis naturally suggests a two-factor pricing model for each country. The factors are nontraded and traded *consumption* growths. We note that in the current setting of exchange economies, the nontraded output is essentially the nontraded consumption and thus is largely internalized within the country. Consequently, shocks in nontraded output are perceived as a consumption risk and the corresponding factor price is necessarily positive.[4] Using carry trade portfolios as test assets, Table 9 reports a statistically significant and positive (two-stage GMM)

[4] In contrast, movements in home traded *consumption* are not necessarily a risk factor to home investors because this consumption is an equilibrium result of several offsetting effects in the model.

estimate of 30 basis points for the nontraded consumption factor price, from the U.S. investors' perspective.

Related Literature: The current paper contributes to a vibrant strand of international finance literature that attempts to rationalize currency premia as risk-adjusted excess returns. Various risk factors and structural features have been identified, among others, fundamental consumption risks in Lustig and Verdelhan (2007), recursive preferences in Colacito and Croce (2011), habit formations in Verdelhan (2010) and Stathopoulos (2017), economic sizes in Hassan (2013), commodity versus final good productions in Ready et al. (2017), geopolitics and trade gravities in Lustig and Richmond (2018), and trade network centralities in Richmond (2019). This paper pursues the nontraded output growth risk on the basic intuition that such a risk is internalized by, and impactful to, the host country's asset prices. We show that nontraded output growth risk matters, both on its own as well as on its coupling to economy's size, to interest rates, exchange rates, and currency premia. In the difference with Hassan (2013), who studies the SDF associated the numeraire of the common traded consumption good, we investigate the country-specific SDF associated the numeraire of country's consumption basket. As a result, our risk premia reveal the pricing of nontraded output risk from the local investors' perspective, for whom the nontraded output risk matters the most. The pricing effects of output nontradability have been studied in early work by Stulz (1987), Stockman and Dellas (1989), Backus and Smith (1993), and Zapatero (1995), and the pricing effects of trade costs by Dumas (1992) and Sercu et al. (1995). We build on and extend these analyses to incorporate and motivate partial nontradability as variable trade frictions. No-arbitrage factor pricing and strategies of currency markets are investigated in many papers, including Brunnermeier et al. (2009) and Burnside et al. (2011) on skewness risks, Lustig et al. (2011) on SDF paradigm, Menkhoff et al. (2012) on volatility risks, Della Corte et al. (2016) on global imbalance risks, Maurer et al. (2017) on principal component (PCA) factors. Our paper pursues currency portfolio strategies based on nontraded output growth risks. We demonstrate empirically that the portfolios' returns sort with the risk exposure, and the implied price of nontraded output growth risk (i.e., factor price) is significant in these portfolios.

The paper is structured as follows. Section 2 presents a basic international asset pricing model with traded and nontraded goods. Section 3.1 analyzes implications on interest rates, both with and without trade frictions. Section 3.2 analyzes implications on currency strategies. Section 4 presents empirical tests on the pricing effects of nontraded output growth risks in interest rate and currency markets. Section 5 concludes. Appendices A and B describe sources and treatment of data. Appendix C presents technical derivations. Online Appendix D discusses extensions with multiple traded goods, arbitrary trade configuration and incomplete financial markets.

2 Basic Model

The basic model of the world endowment economy consists of K countries, in which each country engages in trade of a single consumption good with every other country. Each country also consumes a country-specific nontraded good, which can be consumed only in that country. The model features real quantities, which in later sections will be paired with their empirical counterparts after we factor out inflation components. We concentrate on the consumption risk in this paper and abstract from production aspects. The countries are endowed with country-specific streams of these traded and respective nontraded goods, whose endowments (interchangeably, outputs) $\{\Delta_T^H, \Delta_N^H\}$ follow diffusion processes

$$d\log\Delta_T^H = \mu_T^H dt + \sigma_T^H dZ_T^H; \qquad d\log\Delta_N^H = \mu_N^H dt + \sigma_N^H dZ_N^H; \qquad H \in \{1, \ldots, K\}.$$

Throughout, the superscript H denotes the country, subscripts T, N the traded and nontraded goods, and Z_T^H and Z_N^H the standard (possibly multi-dimensional) Brownian motions characterizing the country-specific supply shocks of the traded and nontraded sectors. For simplicity, we also omit time index t when possible. Let us first assume that the traded good is shipped without frictions around the globe, and financial market is complete.[5] The market clearing mechanism then simply enforces that traded good outputs from all countries are pooled into the *global* (aggregate) traded endowment Δ_T that determines the traded good supply dynamics

$$\Delta_T \equiv \sum_{H=1}^{K} \Delta_T^H; \qquad d\log\Delta_T \equiv \mu_T dt + \sigma_T dZ_T.$$

Each country features a representative agent who maximizes the expected utility weighted over traded and nontraded per-capita consumptions $C \equiv \{C_T, C_N\}$. The period utilities have the following standard form

$$U^H(C^H, t) = e^{-\rho t}\frac{(C^H)^{1-\gamma}}{1-\gamma} = e^{-\rho t}\frac{1}{1-\gamma}\left[\omega_T (C_T^H)^{1-\epsilon} + \omega_N (C_N^H)^{1-\epsilon}\right]^{\frac{1-\gamma}{1-\epsilon}}; \quad \omega_T + \omega_N = 1, \quad (1)$$

[5] We reinstate the transportation cost in the next section. Incomplete markets are considered in Appendix D.2.

where ρ denotes the time discount factor. This utility features constant intertemporal elasticity of consumption of $\frac{1}{\gamma}$, and constant elasticity of substitution between traded and nontraded goods of $\frac{1}{\epsilon}$. They satisfy the conditions $\gamma > 0$, $\epsilon > 0$. Countries may have different tastes $\{\omega_T, \omega_N\}$ for traded and non traded goods to model possible home biases in consumption. Their normalization is purely conventional.

Equilibrium Consumption Allocation

We consider the competitive equilibrium in which each country's representative agent takes prices as given and dynamically allocates consumptions and savings (i.e., investments in financial assets) to maximize her expected utility subject to the budget constraint. Market clearing then determines goods and assets prices. Because markets are complete, equilibrium consumption allocations across countries can be conveniently characterized as a static optimization at each date

$$\max_{\{C_T^H\}_{H=1}^K} \sum_{H=1}^K \Lambda^H \frac{e^{-\rho t}}{1-\gamma} \left[\omega_T (C_T^H)^{1-\epsilon} + \omega_N (\Delta_N^H)^{1-\epsilon}\right]^{\frac{1-\gamma}{1-\epsilon}} \quad \text{s.t.} \quad \sum_{H=1}^K C_T^H = \Delta_T.$$

Note that the intra-country market clearings allow us to explicitly replace the nontraded consumptions by the respective nontraded endowments. The $\{\Lambda^H\}_H^K$ are the Pareto weights, and characterized by countries' wealth. In equilibrium, the marginal utilities of traded good are equal across countries

$$\Lambda^H \frac{\partial U^H}{\partial C_T^H} = \Lambda^F \frac{\partial U^F}{\partial C_T^F} \equiv M_T, \qquad \forall H, F \in \{1, \ldots, K\}. \tag{2}$$

In principle, these $(K-1)$ first-order equations together with the traded good's market clearing condition determine the K equilibrium traded consumptions $\{C_T^H\}_{H=1}^K$. In practice, because marginal utilities are nonlinear functions of consumption, the equilibrium allocation is not known in closed form. Instead, we log-linearize this world optimization problem to obtain an approximate but intuitive solution for the sake of analysis. Let the lower-case letters always denote the respective log quantities; $c \equiv \log C$, $\delta_T \equiv \log \Delta_T$, $\delta_N \equiv \log \Delta_N$. The equilibrium log (per-capita) traded consumption is (Appendix C)

$$c_T^H = \delta_T + \frac{1}{\gamma\omega_T + \epsilon\omega_N} \left\{ -\rho t - (\gamma - \epsilon)\omega_N \left(\left[1 - \frac{\Lambda^H}{\Lambda}\right] \delta_N^H - \sum_{F \neq H}^K \frac{\Lambda^F}{\Lambda} \delta_N^F \right) \right\}, \tag{3}$$

where we recall that δ_T is the log *aggregate* traded output. $\Lambda \equiv \sum_{H=1}^{K} \Lambda^K$ is a measure of the global GDP, therefore $\frac{\Lambda^H}{\Lambda}$ the relative GDP size of countries. This consumption allocation was first obtained by Hassan (2013), in a model that employs initial wealth transfers among households, and focuses on the relative GDP size, the hedging and the risk aversion effects. We instead concentrate on *nontraded* output growth risks and their asset pricing aspects. These risks impact asset prices both on its own, reflecting a primary aspect that nontraded outputs are largely internalized by the host countries, and on their coupling with economies' sizes, reflecting a second aspect that nontraded output risks can be partially mitigated by international trades.

Specifically, a country's equilibrium traded consumption is influenced by the substitution of nontraded-traded consumptions and international trades. H adjusts its traded consumption c_T^H to (i) respond to changes in aggregate traded endowment δ_T, (ii) substitute for changes in H's nontraded output δ_N^H, and (iii) respond to changes in trade partner F's traded consumption demand due to F's nontraded endowment shock. The aggregate, but not country-specific, log traded endowment δ_T affects the log traded consumption of every country uniformly as a result of frictionless trading and perfect sharing of the traded output risk. The country-specific nontraded endowments δ_N^H, δ_N^F are coupled with sizes in the equilibrium traded consumptions. In particular, while trade partner F's nontraded output impact is enhanced by F's size (term $\frac{\Lambda^F}{\Lambda}$ in (3)), H's own nontraded output impact is subdued by its size (term $1 - \frac{\Lambda^H}{\Lambda}$). Because fluctuations in *per-capita* traded consumption of a larger trade partner F project a larger impact on the common marginal utility, the size Λ^F amplifies F's nontraded shock impact on the rest of the world.[6] In contrast, a larger home country H requires a larger traded consumption adjustment, i.e., a more challenging transaction with its trade partners.[7] To illustrate, in the limit where $\frac{\Lambda^H}{\Lambda} \to 1$, the dominant economy H consumes nearly the entire global supply of traded output, which is exogenous and thus non-responsive to whatever happens to H's nontraded output.

Quantitatively, the above substitution between nontraded and traded consumptions is as follows. When $\gamma > \epsilon$, country H's traded consumption c_T^H increases with its trade partners' nontraded endowments δ_N^F and decreases with its own δ_N^H. Intuitively, when the elasticity of substitution between traded and nontraded consumptions $\frac{1}{\epsilon}$ is higher than that of intertemporal substitution

[6] We recall that endowment and consumption are per-capita, and hence the marginal utilities of traded good are equalized up to the size factor; $\frac{\Lambda^H}{\Lambda} \frac{\partial U^H}{\partial C_T^H} = \frac{\Lambda^F}{\Lambda} \frac{\partial U^F}{\partial C_T^F}, \forall H, F \in \{1, \ldots, K\}$. This observation seems particularly germane in the situation in 2009-2010, when Europe and the United States are suffering significant downward shocks to their nontraded production.

[7] It has long been observed that small nations get more from and are more affected by international trade than are large countries, other factors equal. This observation adds an additional dimension to this dynamic.

$\frac{1}{\gamma}$, investors are primarily concerned with smoothing consumption over time, and thus are always eager to adjust their traded-nontraded consumption composition to achieve this smoothing. As a result, traded consumption c_T^H responds strongly to nontraded supply shocks. All else being equal, in times of home nontraded surplus ($dZ_N^H > 0$), investors substitute traded consumption ($dc_T^H < 0$) with the home nontraded good that has become relatively cheaper. Similarly, in times of foreign nontraded surplus ($dZ_N^F > 0$), foreign investors demand less, and home investors end up consuming more, traded good ($c_T^H > 0$) due to a global market clearing of the traded good. To formalize these relationships, we assume the following.

Assumption 1 *The elasticity of substitution between the traded and nontraded consumptions is higher than that of the intertemporal substitution,* $\frac{1}{\epsilon} > \frac{1}{\gamma}$.

Stochastic Discount Factors

In consumption-based settings, a country's currency (i.e., numeraire) is associated with its consumption basket, i.e., the lowest-cost consumption bundle that delivers one unit of the country's utility. For the home country, the stochastic discount factor (SDF) prices assets in units of the home currency, and equals H's marginal utility of home-specific consumption aggregator, $M^H = \frac{\partial U^H}{\partial C^H} = e^{-\rho t}(C^H)^{-\gamma}$. The home-specific log SDF, $m^H \equiv \log M^H$, is (Appendix C)

$$m^H = -\rho t - \gamma\omega_T\delta_T - \gamma\omega_N\left[\delta_N^H - \alpha(\gamma-\epsilon)\omega_T\left(1-\frac{\Lambda^H}{\Lambda}\right)\delta_N^H + \alpha(\gamma-\epsilon)\omega_T\sum_{F\neq H}^{K}\frac{\Lambda^F}{\Lambda}\delta_N^F\right], \tag{4}$$

where $\alpha \equiv (\gamma\omega_T + \epsilon\omega_N)^{-1}$ is a weighted elasticity of substitution. Note that M^H characterizes the asset pricing from the home investors' perspectives and in the numeraire of home currency (i.e., home consumption basket). It conceptually differs from the common marginal utility of the traded consumption M_T (2), which prices assets in the numeraire of the common traded consumption good.[8] This distinction is important because nontraded output risks enter these SDFs differently, leading to different corresponding pricing dynamics. A financial asset has different risk-return tradeoff interpretations (i.e., risk premia) in different numeraires because the asset payoff correlates differently with different SDFs. Several important observations about the country-specific SDF M^H are in order.

[8] Hassan (2013) analyzes the asset pricing from the perspective the common SDF M_T associated with traded consumption numeraire. Whereas the current paper employs the country-specific SDF to analyze the pricing effect of nontraded output risks in the country-specific numeraire, i.e., country's currency.

First, the log SDF (4) of any country decreases uniformly with the global supply of the traded good. This property is inherited from a similar property of the equilibrium traded consumption c_T^H (3), and both arise from the feature that traded outputs are fully and frictionlessly internationalized in the model. Second, the home log SDF decreases with its trade partners' nontraded endowments δ_N^F, which also mirrors the behavior of the equilibrium home traded consumption. All else being equal, a surplus in F's nontraded supply prompts country F to curb, and country H to boost, traded consumptions. As a result, H's marginal utility falls. Because this mechanism is mediated by international trades, a larger foreign partner Λ_F projects a stronger impact of its nontraded output risk (in δ_N^F) on the home SDF m^H.

Third, the home SDF moves with own nontraded endowment δ_N^H in two fashions. Indirectly (the second term inside the square brackets in (4)), although H consumes its entire nontraded endowment, H still is able to partially substitute and mitigate its nontraded output shocks by adjusting its traded good's intake. This mitigation arises via international trades of traded outputs, hence is influenced (hindered) by the home economy's size (as a large country faces large consumption adjustment needs, per the discussion below (3)). Directly and importantly (the first term inside the square brackets), movements in the nontraded output δ_N^H are fully reflected in the log SDF m^H. This direct effect arises from the internalization of nontraded output by the host country, and hence does not involve country's size. The nontraded output growth risk goes beyond, and decouples from, economy's size in pricing assets in this direct effect. Combining direct and indirect effects, the nontraded output risk of a larger economy is more impactful to the economy's SDF because the mitigation of this risk (indirect effect) is more limited via international trades.

The following proposition quantifies the impact of home and foreign nontraded output risks on the home SDF.

Proposition 1 *In the current setting of the world economy, although the nontraded output shock of a country is priced by all of its trade partner economies, the home nontraded output risk always influences the home SDF m^H more than it influences the foreign SDF m^H;* $\left|\frac{\partial m^H}{\partial \delta_N^H}\right| > \left|\frac{\partial m^F}{\partial \delta_N^H}\right|$.

That is, the coefficient associated with δ_N^H is invariably larger in m^H than in any other m^F for all size configurations. This is because whereas the latter is entirely mediated by international trades, the former involves the internalization of nontraded output risk δ_N^H by the home country (a direct effect). An immediate consequence of this proposition is that a positive change in home nontraded endowment (respectively, a negative change in foreign nontraded endowment) decreases

m^H more (respectively, increases m^H less) than it does to m^F, and thus widens the SDF differential $(m^F - m^H)$, i.e., the real exchange rate (see also (10)). The asymmetry reported in the above proposition hence helps break the uncovered interest rate parity (UIP) as we discuss in Section 3.2.

3 Nontraded Growth Risks and Asset Prices

The above basic model provides us with an equilibrium framework to analyze effects of nontraded growth risks on interest rates and currency returns in this section. We also extend the basic pricing model to accommodate realistic features of partial tradability and trade frictions to formulate testable implications of nontraded growth risks on asset prices.

3.1 Interest Rates

Before presenting a formal analysis, intuitions suffice to suggest a key role of output nontradability on interest rates without and with trade costs. These interest rates are returns on country-specific bonds that pay off in terms of country-specific consumption baskets. Hence, they are instantaneously risk-free rates from the perspective of investors of respective countries, and intrinsically associated with country-specific SDFs (4).

Trades without Frictions

For simplicity, we first assume that traded goods can be shipped worldwide without costs. All else being equal, when an economy exhibits a higher level of uncertainty, the associated bond offering a sure payoff of one consumption unit becomes more valuable and interest rates drop. Without trade costs, the country-specific traded outputs are indifferently pooled together into the global supply of traded outputs. It is this global supply (but not the country-specific supplies of traded outputs) that matters for every country's interest rate. The more volatile the *global* traded output, the lower interest rates in *all* countries. Thus what causes interest rates to differ across countries should have primarily with nontraded outputs. Volatile nontraded outputs either at home or abroad act to lower home interest rates. A foreign trade partner F with volatile nontraded output transmits this volatility to home country H by inducing frequent traded consumption adjustments. The larger country F is, the stronger is the induction impact, and the more H's interest rate r^H decreases with F's nontraded output volatility. In contrast, the larger home country H is, the less trading room it finds to mitigate H's nontraded output volatility. Consequently, although r^H decreases with own nontraded output volatility, such an inverse relationship is weaker when H is larger.

These intuitions are formalized by examining the expression of the interest rate[9]

$$\begin{aligned} r^H &= \rho + \gamma\omega_T\mu_T - \frac{1}{2}\gamma^2\omega_T^2\sigma_T^2 + \alpha\gamma(\gamma-\epsilon)\omega_T\omega_N \sum_{F=1}^{K} \frac{\Lambda^F}{\Lambda}\mu_N^F - \frac{1}{2}\alpha^2\gamma^2(\gamma-\epsilon)^2\omega_T^2\omega_N^2 \sum_{F=1}^{K} \frac{(\Lambda^F)^2}{(\Lambda)^2}(\sigma_N^F)^2 \\ &+ \alpha\gamma\epsilon\omega_N\mu_N^H - \frac{1}{2}\alpha^2\gamma^2\epsilon^2\omega_N^2(\sigma_N^H)^2 - \alpha^2\gamma^2\epsilon(\gamma-\epsilon)\omega_T\omega_N^2\frac{\Lambda^H}{\Lambda}(\sigma_N^H)^2. \end{aligned} \tag{5}$$

Expected endowment growth rates μ's contribute to increase risk-free rates via the intertemporal consumption smoothing effect. Endowment growth volatilities σ's act to suppress risk-free rates via precautionary savings effect. Terms containing coefficients $(\gamma-\epsilon)\omega_T$ arise from traded consumption sharing and adjusting, wherein ω_T characterizes investors' affection for the traded good and $(\gamma-\epsilon)$ the nontraded shock spill-over to the traded sector by the substitution of two consumption goods. Interestingly, the first five terms (i.e., all terms in the first line of (5)) of risk-free rates are identical across countries. Therefore, nontraded output risks drive the interest rate differential[10]

$$r^H - r^F = \alpha\gamma\epsilon\omega_N\Delta\mu_N - \frac{1}{2}\alpha^2\gamma^2\epsilon^2\omega_N^2 - \alpha^2\gamma^2\epsilon(\gamma-\epsilon)\omega_T\omega_N^2\left(\frac{\Lambda^H}{\Lambda}(\sigma_N^H)^2 - \frac{\Lambda^F}{\Lambda}(\sigma_N^F)^2\right), \tag{6}$$

The home nontraded output risk $(\sigma_N^H)^2$ impacts the home interest rate r_H both on its own and on its coupling with the host economy's size $\frac{\Lambda^H}{\Lambda}$. The overall effect is that, the home nontraded risk is more severe, making home-specific bond more valuable and lowering the home interest rate, when the home economy is larger. This is because a larger home economy finds it more difficult to mitigate its own nontraded output risk, an economic feature also found in the country-specific SDF m^H (4).

Above findings paint two very different pictures for the implication of traded and nontraded growth risks on risk-free rates. Below, we formulate a testable hypothesis highlighting this difference. The actual tests and supportive evidences are presented in Section 4.2.

Hypothesis 1 *All else being equal, the impact of country-specific nontraded output growth risk on home interest rate dominates that of the country-specific traded output growth risk.*

[9]This expression arises from the drift of SDF, $r^H = \frac{1}{dt}\left(-E_t\left[dm^H\right] - \frac{1}{2}\mathrm{Var}_t\left[(dm^H)^2\right]\right)$ (4). For simplicity, we assume that country-specific nontraded outputs are pairwise uncorrelated, and uncorrelated with aggregate traded outputs (extensions are relaxed in Appendix D).

[10]In (6), $\Delta\mu_N$ and $\Delta(\sigma_N)^2$ denote respectively the differential of mean and variance of nontraded output growths across economies H and F.

The key intuition underlying this hypothesis is the diversification of traded output risks. To see this, we concentrate on the explicit contributions of country-specific traded output volatilities σ_T^H to the interest rate (5), omitting terms unrelated to these volatilities,

$$r^H = \# - \frac{1}{2}\gamma^2\omega_T^2\sigma_T^2 = \# - \frac{1}{2}\gamma^2\omega_T^2\frac{1}{dt}\left(\sum_{H=1}^{K}\frac{\Delta_T^H}{\Delta_T}\sigma_T^H dZ_T^H\right)^2.$$

Clearly, the contribution of country-specific traded shocks dZ_T^H is suppressed by the share of a country's traded output in the world $\frac{\Delta_T^H}{\Delta_T}$, as country-specific traded outputs are pooled into the aggregate global output thanks to the tradability of these outputs.[11]

Costly Trades

The previous section's results are derived based on two assumptions, namely, goods are either perfectly traded or nontraded, and trades are frictionless. Consequently, traded output growth risks are aggregated and diversified across borders, giving rise to Hypothesis 1 above. The introduction of trade costs in this section aims to relax both of these simplifications. In particular, the concept of (partial) *tradability* arises naturally via a regulation of trade cost. A traded good can become a nontraded good when trade cost is sufficiently high. The tradability is the key to bringing our model to the data in Section 4.2.

To model the frictions in trades, we adopt the "iceberg transport cost" approach of Samuelson (1954), Dumas (1992) and Sercu et al. (1995). In this modeling approach, the commodity trade is not perfect because only a fraction of $\frac{1}{1+\theta}$ of the original traded good that leaves the exporting country arrives at the importing country, and the remainder "melts" along the way as a result of trade friction. To simplify the exposition, we first consider a single good shared by two countries $\{H, F\}$ of similar sizes.[12] Parameter θ characterizes the trade cost (i.e., non-tradability) of the good, and helps regulate the amount of good being traded (import and export) between countries.[13]

The linearity in transport costs is a key modeling advantage to keep market completeness intact without further assumption. Consequently, the equilibrium is obtained by solving the static world

[11] Tian (2018) makes a related observation that a country's traded consumption growth should be less volatile than the country's traded output growth due to the diversification in the traded good market.

[12] It is straightforward to add transportation costs to the setting of the previous section to have perfectly traded, partially traded and nontraded goods at the same time.

[13] We drop the subscripts T, N throughout this subsection, due to partial tradability nature of the good.

optimization subject to appropriate global resource constraints

$$\max_{\{C_H^H, C_F^H, C_H^F, C_F^F\}} U^H(C^H) + U^F(C^F) \equiv e^{-\rho t}\left[\frac{(C_H^H + C_F^H)^{1-\gamma}}{1-\gamma} + \frac{(C_H^F + C_F^F)^{1-\gamma}}{1-\gamma}\right]$$
$$\text{s.t.} \quad C_H^H + (1+\theta)C_H^F = \Delta^H; \quad C_H^F \geq 0; \quad C_F^F + (1+\theta)C_F^H = \Delta^F; \quad C_F^H \geq 0,$$

where $C^H = \{C_H^H, C_F^H\}$ are home consumption components that originate from home and foreign outputs, respectively (the foreign counterpart is $C^F = \{C_H^F, C_F^F\}$). Thus, C_F^H is the import by H, which derives from the original amount $(1+\theta)C_F^H$ exported from F. Similarly, C_H^F is the import by F and derives from the original amount $(1+\theta)C_H^F$ from H. Countries desire to trade to share their output risks. However, the transport cost may limit the risk sharing. Intuitively, if the cost outweighs the benefit of risk sharing, countries opt not to trade and instead fully internalize their endowment shock; $C_H^F = C_F^H = 0$. To determine the conditions for commodity market no-trade, assume these conditions are currently not met and that trades take place. Because the shipping incurs a cost, the imported good is always more expensive than the locally endowed good, and countries always deplete their endowed resource before reaching out to the imported resource. In other words, conditional on trades taking place, there are two mutually exclusive alternatives:

case 1: H imports, F exports, $\quad C_H^H = \Delta^H; \; C_F^H > 0; \; C_H^F = 0; \; C_F^F < \Delta^F,$

case 2: H exports, F imports, $\quad C_H^H < \Delta^H; \; C_F^H = 0; \; C_H^F > 0; \; C_F^F = \Delta^F.$

By symmetry, it suffices to study case 1. The two FOCs associated with non-binding constraints and the market clearing condition for the home-endowed good establish the remaining equilibrium consumption allocations (i.e., apart from the binding constraints $C_H^H = \Delta^H$, $C_H^F = 0$)

$$C_F^H = \frac{\Delta^F - (1+\theta)^{\frac{1}{\gamma}}\Delta^H}{(1+\theta) + (1+\theta)^{\frac{1}{\gamma}}}; \qquad C_F^F = \frac{(1+\theta)^{\frac{1}{\gamma}}\left[\Delta^F + (1+\theta)\Delta^H\right]}{(1+\theta) + (1+\theta)^{\frac{1}{\gamma}}}. \tag{7}$$

Trades require net positive home import $C_F^H > 0$, and commodity market no-trade prevails otherwise. We analyze these two regimes in turn.

No-trade regime: Combining cases 1 and 2 yields the following no-trade condition for the commodity market:

$$\text{No-trade conditions:} \qquad (1+\theta)^{-1} < \left(\frac{\Delta^H}{\Delta^F}\right)^{\gamma} < (1+\theta).$$

Clearly, either costly transport (large θ), or similar outputs ($\frac{\Delta^H}{\Delta^F} \approx 1$), or low risk aversion (small

γ), discourages countries to trade to share risk, and thus enforces the commodity market no-trade. In this case, the good effectively becomes a nontraded good in both countries. A country's bond does not hedge others' shocks, and the risk-free rate solely reflects the country's own output risk. For each country, the nontraded output volatility is the only risk that matters here.

Costly trade regime: In contrast with the no-trade regime, when friction is moderate and home and foreign outputs are sufficiently different, countries choose to trade to share output risk. Without loss of generality, we continue with case 1 above, in which home is the importing country (or $C_F^H > 0$). Conditional on this being the case, $(1+\theta)^{-1} > \left(\frac{\Delta^H}{\Delta^F}\right)^\gamma$, the home unambiguously curbs its imports when transaction cost increases (C_F^H (7) decreases in θ when $C_F^H > 0$). However, interestingly, the inverse holds for the exporting country F for all realistic values of transport cost and risk aversion. Contingent on trades taking place, the foreign country actually boosts its export $(1+\theta)C_F^H$ when θ increases to compensate for the increasing loss in the transportation.[14] This is because, when home investors are risk averse, their net import C_F^H decreases less than linearly with the transport cost.

As long as trades take place, regardless of their "iceberg-melting" friction, marginal utilities are equalized across countries ($\frac{\partial U^H}{\partial C^H} = (1+\theta)\frac{\partial U^F}{\partial C^F}$), as are the interest rates. We focus on the precautionary savings effect, and consider the associate term in interest rates,

$$r^H = r^F = \# - \frac{1}{2}\gamma(\gamma+1)\frac{(1+\theta)^2(\Delta^H)^2(\sigma^H)^2 + (\Delta^F)^2(\sigma^F)^2}{[(1+\theta)\Delta^H + \Delta^F]^2}. \tag{8}$$

As the transport cost θ increases, interest rates become increasingly sensitive to home output shocks and decreasingly sensitive to foreign output shocks; $\frac{\partial^2|r|}{\partial\theta\partial|(\sigma^H)^2|} > 0$, $\frac{\partial^2|r|}{\partial\theta\partial|(\sigma^F)^2|} < 0$. These behaviors, when combined with the earlier findings that $\frac{\partial C_F^H}{\partial\theta} < 0$ and $\frac{\partial\left[(1+\theta)C_F^H\right]}{\partial\theta} > 0$, precisely support our key *thesis* that when shocks are of a more nontraded nature (i.e., θ increases), they matter more to the country's asset prices. From the importing country H's perspective, a surge in trade cost θ coincides with a reduction in trades as its imports C_F^H drop. At the same time, the impact of the country's own volatility σ^H on its interest rate r^H increases while the impact of foreign volatility σ^F on r^H decreases. Likewise, from the exporting country F's perspective, a surge in trade cost coincides with a boost in trades as its export $(1+\theta)C_H^F$ increases. At the same time, the impact of its own volatility σ^F on its interest rate r^F decreases, whereas the impact of partner's volatility σ^H

[14] $\frac{\partial}{\partial\theta}\left[(1+\theta)C_F^H\right] = \frac{\gamma-1}{\gamma}(1+\theta)^{\frac{1-2\gamma}{\gamma}}\Delta^F - \frac{\gamma-1}{\gamma}(1+\theta)^{\frac{2-2\gamma}{\gamma}}\Delta^H - \frac{1}{\gamma}(1+\theta)^{\frac{1-\gamma}{\gamma}}\Delta^H - \frac{1}{\gamma}(1+\theta)^{\frac{2-2\gamma}{\gamma}}\Delta^H$. For all realistic values of γ and θ, the last two terms are negligible compared with the second term. Then, the trade condition $C_F^H > 0$ immediately implies that $\frac{\partial}{\partial\theta}\left[(1+\theta)C_F^H\right] > 0$.

on r^F increases. Overall, the variation in trade frictions implies a structural relationship between nontradability and domestic asset prices. A refined version incorporating partial tradability of Hypothesis 1 above is

Hypothesis 2 *All else being equal, a country-specific output growth volatility impacts the home risk-free rate more when the output is less tradable.*

The generalization of this hypothesis to multiple-country setting is straightforward. Section 4.2 tests this hypothesis by employing different empirical measures of nontradability.

3.2 Carry Trade Returns

Let us consider the typical carry trade strategy from the perspective of country H's investors, who sell (i.e., borrow) the home currency and buy (i.e., lend) the foreign currency at t, liquidate and convert all proceeds back to the home currency at $t+dt$. Hence, carry trades are bets on exchange rates, earning premia as rewards for bearing the exchange rate risk. Let the spot exchange rate S_t be the quantity of foreign currency that buys one home currency unit at t. In the current complete market setting, the exchange rate equals the SDF ratio, $S_t = \frac{M_t^H}{M_t^F}$. The realized and expected returns, in excess of the home interest rate, of the above carry trade strategy respectively are,

$$\begin{aligned} XR_{t+dt}^{-H,+F} &= \tfrac{1}{dt}\left[\tfrac{M_{t+dt}^F}{M_{t+dt}^H}(1+r_t^F dt)\tfrac{M_t^H}{M_t^F} - (1+r_t^H dt)\right], \\ E_t\left[XR_{t+dt}^{-H,+F}\right] &= -\tfrac{1}{dt}Cov_t\left[dm^H, dm^F - dm^H\right]. \end{aligned} \tag{9}$$

The Underlying Risk

Reassuringly, the carry trade expected excess return (9) is the exchange rate risk premium priced by the home SDF, $E_t\left[XR_{t+dt}^{-H,+F}\right] = -Cov_t\left[\frac{M_{t+dt}^H}{M_t^H}, dS_{t+dt}^{-1}\right]$. The consumption fluctuations contribute to the currency premium precisely because they perturb both SDFs m^H, m^F via direct consumption and trade. Here our discussion is readily carried over from the previous section's analysis on SDFs. Because traded shocks spread uniformly to all countries, they do not affect exchange rates, and are not priced in the carry trades. This leaves fluctuations in nontraded outputs as the sole source of currency risk and return in the current setting. Indeed, the log exchange rate is implied from (4)

$$d\log S_t = dm^H - dm^F = \#dt + \gamma\alpha\epsilon\omega_N\left(\sigma_N^H dZ_N^H - \sigma_N^F dZ_N^F\right). \tag{10}$$

On one hand, an adverse foreign nontraded shock $dZ_N^F < 0$ makes F's nontraded good scarce and suppresses the real exchange rate S (i.e., foreign currency appreciates), and therefore $m^F - m^H$ surges (see also Proposition 1). On the other hand, $dZ_N^F < 0$ also forces F to consume more and H to consume less traded goods, and m^H surges. That is, the long bet on the foreign currency pays off well when home investors highly value consumption. This carry trade strategy hence is a *hedge* against the foreign nontraded risk. All else being equal, the expected return $E_t\left[XR^{-H,+F}\right]$ is lower when the foreign nontraded risk is more significant in equilibrium.

In contrast, an adverse home nontraded shock $dZ_N^H < 0$ directly boosts m^H. Moreover, H demands more traded goods to substitute for its loss in nontraded goods, leaving F with less traded consumptions. Hence, m^F increases but by a lesser extent than m^H does. Consequently, $m^F - m^H$ drops because the real exchange rate S increases (i.e., home currency appreciates). That is, the long bet on the foreign currency pays off poorly when home investors highly value consumption. This carry trade strategy hence is *not* a hedge against the home nontraded risk. All else being equal, the expected return $E_t\left[XR^{-H,+F}\right]$ is higher when the home nontraded risk is more significant. The overall expected profit (or loss) of the carry trade is determined by whether home (or foreign) nontraded risk dominates, as seen quantitatively in the following result.

Proposition 2 *The expected carry trade excess return to U.S. investors is*

$$E_t\left[XR_{t+dt}^{-H,+F}\right] = \alpha^2\gamma^2\epsilon\omega_N^2 \left\{\left[\epsilon + (\gamma-\epsilon)\omega_T\frac{\Lambda^H}{\Lambda}\right](\sigma_N^H)^2 - (\gamma-\epsilon)\omega_T\frac{\Lambda^F}{\Lambda}(\sigma_N^F)^2\right\}, \tag{11}$$

where $\alpha \equiv (\gamma\omega_T + \epsilon\omega_N)^{-1}$ is a weighted elasticity of consumption substitution. Consequently, the carry trade strategy offers an expected profit when the home nontraded output risk coupled with the home size dominates,

$$\left[\epsilon + (\gamma-\epsilon)\omega_T\frac{\Lambda^H}{\Lambda}\right](\sigma_N^H)^2 > (\gamma-\epsilon)\omega_T\frac{\Lambda^F}{\Lambda}(\sigma_N^F)^2.$$

The carry trade is a good (bad) hedge against the foreign (home) nontraded output growth risk. When home nontraded risk dominates, $(\sigma_N^H)^2 \gg (\sigma_N^F)^2$, this strategy is risky and offers high expected returns $E_t\left[XR_{t+dt}^{-H,+F}\right] > 0$, and vice versa. However, when $(\gamma-\epsilon)\omega_T$ is positive but small, investors are not enthusiastic about substituting nontraded for traded consumption goods. This weakens the trade effect, makes home nontraded output risk even worse to home investors, and compensates them with higher expected returns.

Our analysis not only provides a risk-based explanation for widely-practiced carry trade strategies, it also suggests the following currency bet based on the nontradability aspects of output growth risk revealed in (11),

Hypothesis 3 *Borrowing currencies of economies with volatile nontraded sectors and large sizes while lending currencies of economies with stable nontraded sectors and small sizes generate positive expected returns.*

We examine empirically the merits of this macro-based strategy in Section 4.3. A reflection on the behaviors of risk-free rates and carry trade returns above indicates that the nontraded growth risk is a factor behind the violation of uncovered interest rate parity (UIP). In the regression-based approach, this violation is an empirical regularity in which appreciating currencies also tend to be associated with increasing interest rates (e.g., Hansen and Hodrick (1980) and Fama (1984)). In the portfolio-based approach, this violation is usually associated with the premia of currency strategies based on the interest rate sorting (e.g., Lustig and Verdelhan (2007)). Hassan and Mano (2019) clarify the distinction of the two approaches, and attribute them respectively to cross-time and cross-currency movements in data. The nontraded growth risk conceptually falls into the cross-currency line of analysis. When the home country has volatile nontraded sector (σ_N^H large), home risk-free bonds are valuable as a safe asset, and the home interest rates is low (r^H small). At the same time, expected carry trade returns are high because these strategies are exposed to the significant home nontraded risk. When the foreign nontraded sector is stable (σ_N^F small), the foreign interest rate is high (r^F large), *and* expected carry trade returns to home investors are also high (Proposition 2). We can also conceptually disentangle the nontraded growth risk's direct effect on carry trade premia (beyond its influence on the interest rate differential) by using (6) to rewrite return (11) as

$$E_t\left[XR_{t+dt}^{-H,+F}\right] = r^F - r^H + \alpha\gamma\epsilon\omega_N\left(\mu_N^H - \mu_N^F\right) + \frac{1}{2}\alpha^2\gamma^2\epsilon^2\omega_N^2\left[(\sigma_N^H)^2 + (\sigma_N^F)^2\right].$$

Evidently, both home and foreign nontraded output volatilities (last terms) contribute to the strategy's premium, and these contributions are not mediated by economies' sizes (instead, sizes are coupled with nontraded growth risks and contribute to the interest rate (5) part of the above premium).[15] All in all, nontraded output risks, originated from either home or abroad, rationalize the

[15] Gabaix and Maggiori (2015) consider financial frictions of limited risk-bearing capacities of intermediaries whose balance sheets are subject to capital flow shocks, and derive the impact of traded (and nontraded) consumptions on the

violation of UIP.

Linear Factor Analysis: Theory

Our finding that country-specific traded and nontraded shocks are priced differently in asset markets naturally indicates a simple linear-factor pricing model in which the risk factors are *country-specific* traded and nontraded consumption growths.

$$f_T^H = \frac{dC_T^H}{C_T^H}; \qquad f_N^H = \frac{dC_N^H}{C_N^H}.$$

We employ carry trade portfolios as test assets to evaluate this pricing model conceptually (below) and empirically (Section 4.3.2).

We consider carry trade return strategies of borrowing home and lending foreign currency. The factor analysis starts with the *unconditional* consumption-based Euler equation for this carry trade return,[16]

$$E\left[\frac{M_{t+dt}^H}{M_t^H} XR_{t+dt}^{-H,+F}\right] = 0 \Longrightarrow E\left[XR_{t+dt}^{-H,+F}\right] = -\frac{1}{dt} Cov\left[dm_{t+dt}^H, XR_{t+dt}^{-H,+F}\right], \tag{12}$$

where $dm_{t+dt}^H \equiv \log M_{t+dt}^H - \log M_t^H$ (4). Because the home consumption is made of both traded and nontraded components, the log-linearized SDF (4) immediately implicates a linear two-factor pricing model of the carry trade from the home perspective

$$E\left[XR_{t+dt}^{-H,+F}\right] = -Cov\left[b_T f_{T,t+dt}^H + b_N f_{N,t+dt}^H, XR_{t+dt}^{-H,+F}\right] \tag{13}$$

$$\begin{bmatrix} b_T \\ b_N \end{bmatrix} = \begin{bmatrix} -\gamma\omega_T \\ -\gamma\omega_N \end{bmatrix}; \quad \begin{bmatrix} f_T^H \\ f_N^H \end{bmatrix} = \begin{bmatrix} \frac{dC_T^H}{C_T^H} \\ \frac{dC_N^H}{C_N^H} \end{bmatrix} = \begin{bmatrix} d\delta_T - \alpha(\gamma - \epsilon)\omega_N \left(d\delta_N^H - \sum_{F=1}^K \frac{\Lambda^F}{\Lambda} d\delta_N^F\right) \\ d\delta_N^H \end{bmatrix},$$

real exchange rate. Our model does not feature financial frictions, and generates the impact of traded consumptions on the exchange rate through their equilibrium adjustments to nontraded (and traded) output shocks.

[16] The unconditional Euler equation (12) follows from its conditional version $E_t\left[\frac{M_{t+dt}^H}{M_t^H} XR_{t+dt}^{-H,+F}\right] = 0$ by taking the unconditional expectation of both sides (applying the law of iterated expectation), $E\left[\frac{M_{t+dt}^H}{M_t^H} XR_{t+dt}^{-H,+F}\right] = 0$, or

$$E\left[\frac{M_{t+dt}^H}{M_t^H}\right] E\left[XR_{t+dt}^{-H,+F}\right] = -Cov\left[\frac{M_{t+dt}^H}{M_t^H}, XR_{t+dt}^{-H,+F}\right].$$

The carry trade excess return is of order $\mathcal{O}(dt, dZ_{t+dt})$: $XR_{t+dt}^{-H,+F} = \#dt + \#dZ_{t+dt}$, and the SDF growth is of order $1 + \mathcal{O}(dt, dZ_{t+dt})$: $\frac{M_{t+dt}^H}{M_t^H} = 1 + \#dt + \log\frac{M_{t+dt}^H}{M_t^H} \equiv 1 + \#dt + dm_{t+dt}^H$. Therefore, up to the order $\mathcal{O}(dt)$, the left-hand side of above equation is $E\left[XR_{t+dt}^{-H,+F}\right]$, while the right-hand side is $-Cov\left[dm_{t+dt}^H, XR_{t+dt}^{-H,+F}\right]$, implying (12).

where $\{f_T^H, f_N^H\}$ are risk factors, and $\{b_T, b_N\}$ the associated loadings on these factors. This linear-factor model prices risks from the home investors' perspective, and can also be cast in its beta-pricing version

$$E\left[XR_{t+dt}^{-H,+F}\right] = \lambda_T^H \beta_T^{H,F} + \lambda_N^H \beta_N^{H,F}, \tag{14}$$

$$\begin{bmatrix} \lambda_T^H \\ \lambda_N^H \end{bmatrix} \equiv \left[Cov(\vec{f}^H, \vec{f}^H)\right] \begin{bmatrix} -b_T \\ -b_N \end{bmatrix}; \quad \begin{bmatrix} \beta_T^{H,F} \\ \beta_N^{H,F} \end{bmatrix} \equiv \left[Cov(\vec{f}^H, \vec{f}^H)\right]^{-1} \begin{bmatrix} Cov(f_T^H, XR^{-H,+F}) \\ Cov(f_N^H, XR^{-H,+F}) \end{bmatrix},$$

where $\left[Cov(\vec{f}^H, \vec{f}^H)\right]$ denotes the 2×2 variance-covariance matrix of the factors $\{f_T^H, f_N^H\}$. As β are slope coefficients of returns linearly regressed on the risk factors, the magnitude of β quantifies the exposures of investment strategies to the two risk factors. In contrast, factor prices $\{\lambda_T^H, \lambda_N^H\}$ are the rewards (in the form of expected returns) to bear one notional unit of corresponding risk (i.e., as if $\beta = 1$), which are independent of assets.

How exactly are risks embedded in asset payoff priced by the home investors? In a basic risk-return tradeoff picture, any factor that moves asset payoff and home marginal utility in opposite directions is perceived as risk (these assets pay poorly when investors highly value the payoff). Hence, the associated factor price is positive, and vice versa. We begin with the home nontraded consumption growth risk. The substitution of factors f's and loadings b's from (13) into (14) yields the following testable result concerning the nontraded factor price.

Proposition 3 *The factor price associated with nontraded consumption growth risk is unambiguously positive,*

$$\lambda_N^H = \alpha\gamma\omega_N \left[\epsilon + (\gamma - \epsilon)\omega_T \frac{\Lambda^H}{\Lambda}\right] (\sigma_N^H)^2 > 0 \quad \forall H. \tag{15}$$

That is, fluctuations in the domestic nontraded consumption growth always pose as a risk to home investors in all countries.

Section 4.3.2 empirically valuates this two-factor pricing model and obtains statistically significantly positive estimate for the U.S. nontraded consumption growth factor price. The nontraded factor price λ_N^H (15) is country-specific. It involves only the volatility σ_N^H because the nontraded output can only be consumed domestically. As smaller economies are able to manage this risk better by more flexibly adjusting their smaller traded consumptions, this risk is less costly for them. Indeed, λ_N^H increases with economy's size Λ^H (15).

We next turn to the home factor price of the traded consumption growth risk. Substituting

(13) into (14) yields

$$\lambda_T^H = \gamma\omega_T(\sigma_T)^2 + \alpha^2\gamma(\gamma-\epsilon)^2\omega_T\omega_N^2 \sum_{F\neq H}^{K} \frac{(\Lambda^F)^2}{(\Lambda)^2}(\sigma_N^F)^2 - \alpha^2\gamma(\gamma-\epsilon)\omega_N^2\left(1-\frac{\Lambda^H}{\Lambda}\right)\left[\epsilon+(\gamma-\epsilon)\omega_T\frac{\Lambda^H}{\Lambda}\right](\sigma_N^H)^2. \quad (16)$$

The traded factor price λ_T^H is also country-specific. But in contrast with λ_N^H, fluctuations in home traded consumption growth are an equilibrium result of several effects. Three terms in (16) signify that a drop in home traded equilibrium consumption is due to either (i) a drop in the global traded output (direct effect), (ii) a drop in trade partners' nontraded outputs (substitution and trade effects), or (iii) a *surge* in the home nontraded output (substitution effect). In particular, nontraded out risks are always enhanced by the associated country's size because these risks matter to the home traded consumption via the substitution of traded and nontraded consumptions. Furthermore, both home (term $(\sigma_N^H)^2$) and foreign (term $(\sigma_N^F)^2$) nontraded output growth risks also affect the price of the home traded factor price λ_T^H in equilibrium. But in difference with λ_N^H (Proposition 3), the sign of λ_T^H depends on the relative magnitudes of global traded and country-specific nontraded output risks (enhanced by countries' sizes). Section 4.3.2 presents the estimates of factor prices for the U.S. and further analysis.

4 Empirical Results

The principal assertion of this paper, motivated by theoretical considerations in preceding sections, is that fluctuations in nontraded output growths are an important risk factor determining asset prices in international markets. This section investigates this assertion empirically and provides supportive evidence. We implement various tests on interest rates and carry trade returns. Our empirical analysis involves OECD countries plus Eurozone (i.e., Economic and Monetary Union, available after 1998). These are more developed economies, whose data series tend to be more complete and of higher quality.[17] Since the model is real while data are nominal, we first factor out the inflation component from empirical quantities, before using them in tests of model's implications. All employed data series are cited in double quotes, and their original sources and other details are

[17]In our notation, before the German reunification in 1990 (and including that year), the Federal Republic of Germany (FRG) is referred to as West Germany. From 1991 onward, the (reunified) Federal Republic of Germany is referred to as Germany.

listed in the data appendix.

4.1 Stylized Facts Concerning Nontraded Output Risk

We identify "construction" and "services" as nontraded sectors in all countries, following the standard classification in the literature (see, e.g., Stockman and Tesar (1995)). Components of services sectors include the following International Standard Industrial Classification of All Economic Activities Revision 4 (ISIC 4) codes: (F) Construction; (G) Wholesale, retail trade and repair; (H) Transportation and storage; (I) Accommodation and food service activities; (J) Information and communication; (K) Financial and insurance activities; (L) Real estate activities; (M) Professional, scientific and technical activities; (N) Administrative and support service activities; (O) Public administration, defence, social security; (P) Education; (Q) Human health and social work activities; (R) Arts, entertainment and recreation; (S) Other service activities; (T) Activities of households as employers; and (U) Activities of extraterritorial organizations and bodies.

To have an idea about the size of nontraded sectors in the economies worldwide, Figure 1 plots the ratio of nontraded output over real GDP, averaged over the period 1970-2016, for all OECD countries plus Eurozone. Output data are from "Aggregate National Accounts: Gross domestic product", and services output are computed as the sum of outputs in all ISIC 4 codes F-U. The figure shows that nontraded outputs constitute a substantial fraction of the total GDP in all OECD countries, ranging from 0.45 (New Zealand) to 0.67 (U.S.) to 0.80 (Luxembourg). Among others, this figure thus re-documents a known fact that services sectors carry a significant weight of the U.S. economy.

To justify the identification of services as a nontraded sector, Table 1 lists the country-specific tradability and size of different service categories for a representative set of 13 OECD countries (see data appendix for classification details). Tradabilities and sizes are averaged over the period 1970-2016. The country-tradability of services is (one half of) the ratio of total exports and imports over the total output of these services by the country (see (19)). The economic size of services is the ratio of total domestic output of these services over the country's GDP. All import, export and output data for each service category is from "OECD Structural Analysis" (STAN), combined with "Trade by Enterprise Characteristics" (TEC) database.[18] Country total GDP is from "Aggregate

[18]STAN 3 (up to 2009) has both trade and output data, STAN 4 (from 2009) has only output data, trade data is moved to TEC. Further information on the matching between STAN 3, STAN 4 and TEC data are provided in the data appendix.

National Accounts: Gross domestic product". The table shows that, whereas the tradabilities and sizes of the same services vary considerably across OECD economies, their tradabilities are indeed small (in the order of few percentage points, and rarely exceeding 15%). In particular, service codes J-N (including information, communication, financial services, and other business activities) are a substantial part of GDP in all countries (ranging from 16.9% for Australia to 46.29% for Luxembourg), yet their tradabilities are very low (ranging from 0.01% for Australia to 1.52% for United Kingdom). Similarly, Table 2 lists the 15 major industries in the U.S., along with their two measures of tradability. The U.S.-specific tradability of an industry is computed using country-specific tradability (19) for the U.S. In the determination of OECD tradability (see (18)), export, import and output are OECD-aggregate quantities. These industry-level macro series are from the STAN and TEC databases. Table 2 shows a marked difference between the tradability of the first 9 industries (manufacturing sector) and the next 6 industries (service sector), which justifies the classification of traded and nontraded goods adopted in the literature as well as in the current paper. The table also shows that country-specific tradabilities do not necessarily and quantitatively coincide with their OECD counterparts because countries are heterogeneous in their consumption and production. For the sake of robustness, our tests presented in the next section will employ both of these tradability measures.

To have a sense of the level of nontraded output risk across countries, Figure 4 plots the volatility of per-capita nontraded output growth for each OECD country. The volatility is computed as the standard deviation of these per-capita nontraded output growth series over the entire period of 1970-2016. Per-capita quantities are computed using the population data from OECD "Aggregate National Account". This figure shows that the degree of fluctuations of nontraded output varies widely across OECD countries. In particular, Switzerland is a relatively small economy among OECD countries, yet its per-capita nontraded output growth is substantially more volatile than other countries (see also Figure 2). While our theoretical consideration (Section 3) indicates that nontraded output growth risks and sizes of economies are important for asset prices, it remains to see how these two factors combined contribute empirically in the cross section of OECD countries.

To have a sense of the level of trade "openness" of OECD countries, Figure 5 plots the ratio of each country's total exports and imports over its GDP (see also (17)), averaged over the period 1970-2016. The figure shows that trade openness is markedly heterogeneous across OECD countries, ranging from 0.19 for the U.S. to 2.21 for Luxembourg. It is known that this ratio can be biased downward for larger economies, and hence a low value of the openness for a country does not necessarily imply high (tariff or non-tariff) obstacles to foreign trade. Rather, the low value of the openness can be a measure of either a weak reliance of domestic producers on foreign supplies and markets or a country's geographic remoteness from potential trading partners. Any of these possible causes are consistent with our notion that the output growth risk of the less open economies is internalized by home countries to a larger extent.

4.2 Interest Rates

In reality, no goods are either perfectly nontraded or perfectly traded. Even if some goods were, macro output series are inevitably subject to measurement errors. Furthermore, costs in trades also affect the structural relation between nontraded output risk and asset prices. In this section, we investigate the empirical relationship between nontraded output volatility and the level of real interest rate across OECD countries, taking into account these practical regularities. We also control for the size of economies to focus on the output risk of various tradability degrees and its effect on real interest rates. Specifically, we devise four tests based on the various classifications of nontradability, in order of increasing sophistication. These regression-based tests involve (i) the closedness of an economy, (ii) the brute-force cutoff dummy of nontradability at the industry level, (iii)the global nontradabilities at the industry level, and (iv) country-specific nontradabilities at the industry level, respectively.

4.2.1 Tests Using Countries' Trade Closedness

The hypothesis to be examined here is that when an economy is exposed more to international trades, its nontraded risk can be better mitigated through trades and the substitution between traded and nontraded consumption. This assertion is motivated by the structural model with trade frictions in a specific form of Hypotheses 1-2 (Section 3.1). The basic regression test of this relationship reads

$$r_t^H = \alpha + \beta_\sigma (\sigma_t^H)^2 + \beta_C \mathcal{C}_t^H + \beta_{\sigma C} (\sigma_t^H)^2 \mathcal{C}_t^H + \beta_x X_t^H + \epsilon_t^H,$$

where σ^H denotes the per-capita GDP growth volatility and X's the various control variables. We adopt the common definition of a country's trade openness $\mathcal{O}^H$ as trade-to-GDP ratio (trade being the sum of export and import), from which also follows the closedness $\mathcal{C}^H$

$$\mathcal{O}^H = \frac{\mathrm{IM}^H + \mathrm{EX}^H}{\mathrm{GDP}^H}; \qquad \mathcal{C}^H = 1 - \frac{\mathrm{IM}^H + \mathrm{EX}^H}{\mathrm{GDP}^H}. \tag{17}$$

Table 3 reports the results associated with this regression. Both national output data and import/export data are from OECD "Aggregate National Accounts". We compute the volatility of per-capita GDP growth over each of five non-overlapping decades (1970-1979, 1980-1989, 1990-1999, 2000-2009, 2010-2016), and the mean of interest rates (dependent variable) over exactly the same periods. Control variables include per-capita GDP mean growth, GDP size (or the ratio of countries' GDP over the aggregate GDP of OECD group), and inflation volatility.[19] The key observation from Table 3 is that the slope coefficients associated with the interaction term (variance $\times$ closedness) are always negative. These coefficients are statistically significant when we take into account the GDP growth (which contributes through the intertemporal smoothing desires of investors), economy size, and inflation risk effects. This negative sign is consistent with the model's central economic rationale that, all else being equal, when a country is less open to trade, the country's output shocks tend to be more internalized and have stronger impacts on lowering country's real interest rate through the precautionary savings mechanism.

4.2.2 Tests Using Multiple Industry Outputs and Nontradability Dummies

Another form of Hypotheses 1-2 (Section 3.1) to be examined in this section is as follows. Controlling for anything else, a country's output growth risk of nontraded industries tends to have a stronger impact on domestic interest rate than its output growth risk of traded industries. Intuitively, this is because country-specific traded risk can be diversified in the global pool of traded goods before it affects prices in any country. The basic regression testing this relationship employs national output data at the industry level. We use binary dummies to classify the nontradability of the industries

$$r^H_{i,t} = \alpha + \beta_\sigma(\sigma^H_{i,t})^2 + \beta_d d_{i,t} + \beta_{\sigma d}(\sigma^H_{i,t})^2 d_{i,t} + \beta_x X^H_{i,t} + \epsilon^H_{i,t},$$

[19] Inflation is computed as the year-to-year percentage change of the consumer price index, and the latter is sourced from OECD "Key Short-term Economics Indicators" database. Furthermore, inflation volatility is computed as standard deviation of the inflation growth. This control variable aims to address the fact that the real asset pricing model does not capture the possible effects from inflation risk.

where $r_{i,t}^H = r_t^H$ is country H's interest rate and thus independent of industry type i, $d_{i,t}$ is nontradability dummy ($d_{i,t} = 1$ for nontraded industries and 0 otherwise, as we explain below). Table 4 reports the results associated with this regression. Countries' real annual industry-level outputs are constructed from the "OECD Structural Analysis (STAN)" database. An industry i is classified as nontraded ($d_{i,t} = 1$) if it belongs to one of the following ISIC 4 classes: 41-43 (construction); 45-56 (Wholesale and retail trade; Repair of motor vehicles and motorcycles; Transportation and storage; Accommodation and food service activities); 58-63 (Information and communication); 64-66 (Financial and insurance activities); 68-82 (Real estate, renting and business activities); 84-99 (Community, social and personal services). Other industries are taken as traded ($d_{i,t} = 0$). We divide the entire time period 1970-2016 into five decade-periods, and the volatility of per-capita output growth for each industry is computed as the respective standard deviation over each period. As before, the control variables include per-capita GDP mean growth, GDP size, and inflation volatility.

The key observation from Table 4 is that the slope coefficients associated with the interaction term (variance $\times$ dummy) are always negative.[20] When we take into account the GDP growth, economy size, and inflation risk effects, these coefficients are statistically negatively significant. The negative sign fits the basic economic intuition that the nontraded output growth risk is more impactful to the economy than that of the traded output. Consequently, the output risk enhances the value of risk-free bonds, and depresses risk-free rate more aggressively when the risk comes from a nontraded industry.

4.2.3 Tests Using Multiple Industry Outputs and Global Nontradabilities

Some industries are not clear-cut traded or nontraded as depicted by a binary dummy of the above regression. In this section, we use continuous-valued global nontradability at industry level to account for this fine distinction. The hypothesis to be examined here is the same as above, namely all else being equal, output risk of nontraded industries matter more to country's interest rate than that of traded industries. The basic regression testing this relationship reads

$$r_{i,t}^H = \alpha + \beta_\sigma (\sigma_{i,t}^H)^2 + \beta_\tau \tau_{i,t} + \beta_{\sigma\tau} (\sigma_{i,t}^H)^2 \tau_{i,t} + \beta_x X_{i,t}^H + \epsilon_{i,t}^H,$$

[20] The first regression without any control variables gives a positive coefficient, but this coefficient is not statistically significant.

where τ_i is a global measure of nontradability of industry i. We adopt the standard definition of tradability as the ratio of OECD aggregate trade over OECD aggregate output of the industry i, and nontradability is the complement to tradability

$$\tau_i = 1 - \frac{\sum_{\text{OECD countries}} [i\text{'s import} + i\text{'s export}]}{2 \times \sum_{\text{OECD countries}} i\text{'s output}}. \tag{18}$$

Table 5 reports the results associated with this regression. Data sources are identical to those employed in the above regression. We use country-specific output series to compute country-specific industry i's growth volatility over each of five decade-periods. We aggregate these series to compute the global tradability and nontradability for each of good i.

The key observation from Table 5 is that the slope coefficients associated with the interaction term (variance $\times$ nontradability) are always negative. When we take into account the GDP growth, economy size, and inflation risk effects, these coefficients are still statistically significant. The negative sign precisely fits the basic economic intuition that as countries mostly internalize their own nontraded shocks, the fluctuations in nontraded industries are more serious risk to the economy than those of the traded ones. Furthermore, output volatility act to lower risk-free rate. Consequently, risk-free rate is more sensitive (and negatively related) to output risk of industries of higher nontradabilities (defined at the OECD level).

4.2.4 Tests Using Multiple Industry Outputs and Country-specific Nontradabilities

In some situation, global measure of tradability does not exactly reflect the tradability of an industry at country level. This happens, e.g., when the trade levels are highly heterogeneous across countries in certain industries. To account for this fine distinction, in this section, we use continuous-valued country-specific nontradability at industry level. The hypothesis to be examined here is the same as above, namely all else being equal, output risk of nontraded industries matter more to country's interest rate than that of traded industries. The basic regression testing this relationship reads

$$r^H_{i,t} = \alpha + \beta_\sigma (\sigma^H_{i,t})^2 + \beta_\tau \tau^H_{i,t} + \beta_{\sigma\tau} (\sigma^H_{i,t})^2 \tau^H_{i,t} + \beta_x X^H_{i,t} + \epsilon^H_{i,t},$$

where τ^H_i is a country-specific measure of nontradability of industry i. We adopt the standard definition of tradability as one half of the ratio of national trade over national output of the industry i, and nontradability τ_i^H is the complement to tradability

$$\tau_i^H = 1 - \frac{[i\text{'s import} + i\text{'s export}] \text{ by country } H}{[2 \times i\text{'s output}] \text{ by country } H}. \tag{19}$$

Table 6 reports the results associated with this regression. Data sources are identical to those employed in the above regression. We use country-specific output series to compute both country-specific industry i's growth volatility over each of five decade-periods and i's country-specific tradability and nontradability.

The key observation from Table 6 is that the slope coefficients associated with the interaction term (variance $\times$ nontradability) are always negative. When we take into account the GDP growth, economy size, and inflation risk effects, these coefficients are still statistically significant. The negative sign precisely fits the basic economic intuition that as countries mostly internalize their own nontraded shocks, the fluctuations in nontraded industries are more serious risk to the economy than those of the traded ones. Furthermore, output volatility lowers risk-free rate via precautionary savings mechanism. Consequently, risk-free rate is more sensitive (and negatively related) to output risk of industries of higher nontradabilities (defined at the country level).

4.3 Carry Trade Returns

The evidences above shows that nontraded risk is an important factor behind national asset returns. Taking a step further, as *international* investment strategies are exposed to country-specific nontraded as well as aggregate traded risks, the strategies' risk premia should reflect the interplay of these risk factors, as well as their interaction with countries' sizes. This is because smaller countries find it easier to adjust their traded consumptions to manage their own nontraded output growth risks. By the same reason, nontraded output growth risks of larger countries exert stronger impacts on their trade partners and international markets. In this section, we investigate the empirical relationship between carry trade expected returns and nontraded output risk of the countries involved. Specifically, we devise two sets of tests which involve (i) evaluating currency portfolios based on one- and two-dimensional sorts along the variance of nontraded output growth, the economy's real size, and their product (Proposition 2) and (ii) constructing nontraded and traded consumption risk factors to price carry trades (Proposition 3). The valuation of all carry trades is exclusively from the perspective of U.S. investors, for whom the returns are in the denomination of U.S. dollars.[21]

[21] Maurer et al. (2019) give an account of the carry trade pricing in different currency denominations.

4.3.1 Currency Portfolios Based on the Nontraded Output Risk and Economy's Size

The theoretical analysis of Section 3.2 (Equation (11)) clearly indicates that controlling for all else, carry trades that borrow U.S. dollar and lend currencies of countries of less volatile nontraded outputs and smaller sizes tend to yield higher expected returns (and vice versa) to U.S. investors. To directly verify this structural relationship, stated in Hypothesis 3 above, we construct currency portfolios based on the variances of nontraded output growths and economic sizes as suggested by the theory. Forming portfolios helps to suppress the noises in individual currency returns, and delivers large and stable return spreads between portfolios by rebalancing (Lustig and Verdelhan (2007)).

We consider carry trade returns from U.S. investors' perspectives. For each country, we identify nontraded output as the outputs on services and construction. This output data is available at quarterly frequency, and sourced from OECD's "Quarterly National Accounts" (QNA) database.[22] For the one-dimensional sort, at the beginning of each quarter t, countries are sorted into four (quartile) portfolios based on the value of one of the following three characteristics:

(i) country-specific per-capita nontraded real output growth variance, or

(ii) relative real GDP size, defined as the ratio of a country's real GDP over the total real GDP of all countries in the sample, or

(iii) their product.

For each country, the variance of the nontraded output growth is computed over the previous fifteen-quarter period, and thus the portfolios are quarterly rebalanced on rolling basis. Portfolio 1 contains countries with lowest value of the characteristic being employed in the sorting, and portfolio 4 the highest. After portfolios' currency compositions are known at the beginning of quarter t, U.S. investors short U.S. dollars and long equally weighted portfolios P ($P \in \{1, 2, 3, 4\}$) of foreign currencies to earn the quarterly returns $XR_{t+1}^{-US,+P}$ realized at the beginning of quarter $t+1$. Following the literature, we assume investors carry out this strategy by entering uncovered long position in 3-month forward exchange contracts of foreign currencies F_i. We denote spot and 3-month forward exchanges rates as USD per unit of currency i at time t by $S_{i,t}$ and $X_{i,t}$. In the data, these rates are given in nominal terms. After factoring out inflation components, the 3-month

[22] To obtain a more extensive historical data, however, U.S. quarterly data are sourced from U.S. Bureau of Economic Analysis. See data appendix for further details.

realized bilateral carry trade real return between currency i and the USD (denominated in USD) is

$$XR_{t+1}^{-US,+F_i} \equiv \ln\left(\frac{S_{i,t+1}}{X_{i,t}}\right) = fd_{i,t} + \Delta s_{i,t+1} + infl_{t,t+1}^{US} - infl_{t,t+1}^{F_i}, \tag{20}$$

where $fd_{i,t} = \ln\left(\frac{S_{i,t}}{X_{i,t}}\right)$ (known at time t) is the forward discount, $\Delta s_{i,t+1} = \ln\left(\frac{S_{i,t+1}}{S_{i,t}}\right)$ the exchange rate growth, and $infl_{t,t+1}^{I}$ country I's inflation, realized during period $(t, t+1)$.[23] The quarterly real returns of the carry trade portfolio is therefore

$$XR_{t+1}^{-US,+P} = \sum_{F_i \in P}^{K_P} \frac{1}{K^P} XR_{t+1}^{-US,+F_i}. \tag{21}$$

The inflation is constructed as log return from each country's quarterly consumer price index (CPI) data (sourced from OECD "Key Short-term Economics Indicators" STEI database), whereas the spot and forward exchange rates data is from Thomson Reuters' Datastream database, which is available from 1976. Finally, the annualized real carry trade returns for each portfolio are obtained by compounding the quarterly counterpart values. We note that because the OECD's QNA data, OECD's STEI data and the Datastream exchange rate data is unbalanced (data start at different times for different countries, see data appendix), not all OECD countries are available at the same time for the purpose of portfolio sorting.

Figure 6 plots the three separate sets of mean annualized returns and Sharpe ratios of four equally weighted and sorted carry trade portfolios. The carry trade strategies borrow USD and lend equally weighted currencies in each portfolio. Each set of results corresponds to one (of the three, listed above) characteristic employed in the sorting. Table 7 reports associated statistics. Notably, the figure shows a monotonically inverse relationship for the set (in green color) of portfolios sorted on the product of nontraded output growth risk and economy's size. Quantitatively, portfolio 1 earns a mean annual real excess return of 2.16% (Sharpe ratio of 23.45%), and portfolio 4 a return of -1.07% (Sharpe ratio of -13.25%) to U.S. investors. Thus a long-short portfolio strategy (long portfolio 1, short portfolio 4) earns mean annual real excess return of 3.23%, and Sharpe ratio of approximately 45%. This empirical inverse relationship is supported by the risk-based rationale that carry trades (borrowing home and lending foreign currencies) are hedges against the foreign

[23] Under the premise of the covered interest rate parity (CIP), the forward discount is equal to the interest rate differential $fd_{i,t} = \ln\left(\frac{R_{i,t}}{R_{US,t}}\right)$, where $R_{US,t}$ and $R_{i,t}$ are 3-month interest rate in the USD and currency F_i. In computing real carry trade return $XR_{t+1}^{-US,+F_i}$ to the U.S. investors, the inflation components in (20) undo nominal gains and losses due to the inflation realized in the U.S. and country F_i during period $(t, t+dt)$. Our results are robust to a replacement of the forward discount by the interest rate differential, i.e., to the CIP assumption.

nontraded output risks (coupled and amplified by the foreign economy's size), offering lower mean returns when the coupling is larger (Hypothesis 3). Currency portfolios sorted on sizes (in blue color) or nontraded output risk (in red color) alone do not exhibit a strictly monotonic pattern, consistent with the theoretical implication (11) that each of these characteristics on its own does not necessarily dominate the currency premia.

To control for sizes, Figure 7 plots mean annualized returns and Sharpe ratios of two-by-two equally weighted and sorted carry trade portfolios. The currencies are sorted into these portfolios along two characteristics (two-dimensional sort), namely, the variance of nontraded real output growth and relative real GDP size (defined above). The carry trade strategies borrow USD and lend equally weighted currencies in each portfolio. Table 8 reports associated statistics. Separately for currencies associated with either small or large size economies, the mean returns of portfolios decrease with the magnitude of nontraded real output growth risk. Similarly, separately for currencies associated with either low or high nontraded real output growth risk, the mean returns of portfolios decrease with size. These patterns provide the robustness for the risk-based rationale of currency premia (Hypothesis 3).

4.3.2 Linear Factor Analysis: Empirics

The theoretical analysis of Section 3.2 suggests another intuitive way to consider nontraded and traded consumption risks as two key pricing factors. From U.S. investors' perspectives, fluctuations in the U.S. traded and nontraded consumption are risks. The payoffs of currency strategies that correlate with these consumptions are priced, resulting in carry trade risk premia. Accordingly, we employ sorted currency portfolios of the previous section as test assets to estimate the factor prices of the two consumption risks. We discuss, in order, the estimation procedure, data, and estimation results.

We empirically identify the U.S. traded and nontraded consumption variations as risk factors for U.S. investors; $f^{US}_{T,t+1} = \frac{C^{US}_{T,t+1} - C^{US}_{T,t}}{C^{US}_{T,t}}$, $f^{US}_{N,t+1} = \frac{C^{US}_{N,t+1} - C^{US}_{N,t}}{C^{US}_{N,t}}$. First, an application of the generalized method of moments (GMM) on Euler pricing equations for test asset (carry trade excess) returns $XR^{-US,+P}_{t+1}$ (21) helps to estimate the factor loadings $\{b_T, b_N\}$.[24] Then follow the factor prices $\{\lambda^{US}_T, \lambda^{US}_N\}$, and portfolios P's exposures $\{\beta^{US,P}_T, \beta^{US,P}_N\}$ to the U.S. traded and nontraded

[24]These Euler pricing equations are $E_t\left[\frac{M^{US}_{t+1}}{M^{US}_t} XR^{-US,+P}_{t+1}\right]=0$, where $dm^{US}_{t+1} \equiv \log \frac{M^{US}_{t+1}}{M^{US}_t}$ is a function of the loadings $\{b_T, b_N\}$: $dm^{US}_{t+1} = b_T f^{US}_{T,t+1} + b_N f^{US}_{N,t+1}$ (13).

consumption growth risk (by replacing the single currency F in (14) by currency portfolio P)

$$\begin{bmatrix} \lambda_T^{US} \\ \lambda_N^{US} \end{bmatrix} = \left[Cov(\vec{f}^{US}, \vec{f}^{US})\right] \begin{bmatrix} b_T \\ b_N \end{bmatrix}; \quad \begin{bmatrix} \beta_T^{US,P} \\ \beta_N^{US,P} \end{bmatrix} = \left[Cov(\vec{f}^{US}, \vec{f}^{US})\right]^{-1} \begin{bmatrix} Cov(f_T^{US}, XR^{-US,+P}) \\ Cov(f_N^{US}, XR^{-US,+P}) \end{bmatrix},$$

Hence, the GMM procedure employed to estimate factor loading b's also estimates factor prices and risk exposures.

Currencies are sorted are sorted into four (quartile) portfolios based on the value of country-specific product of (real per-capita) nontraded output growth variance and relative real GDP size, as explained in the previous subsection. The risk factors f_T^{US}, f_N^{US} are computed as quarter-to-quarter percentage changes of the (real per-capita) U.S. traded and nontraded consumption respectively. The U.S. consumption and CPI series are from U.S. Bureau of Economic Analysis' "Quarterly U.S. consumption expenditures and price indexes". We identify the personal consumption expenditures on "services" as nontraded consumption, and on "goods" as traded consumption (see data appendix for further details).[25] After having constructed the quarterly series of portfolio returns $XR_t^{-US,+P}$ and factors $f_{T,t}^{US}$, $f_{N,t}^{US}$, we employ a two-stage GMM procedure to estimates factor loadings b_T, b_N jointly with the traded and nontraded factor prices λ_T^{US}, λ_N^{US}, and portfolio risk exposures $\beta_T^{US,P}$, $\beta_N^{US,P}$, as mentioned above and also in Menkhoff et al. (2012).[26] Their standard errors are determined from GMM-generated standard errors of factor loading b's and the delta method, as suggested by Burnside et al. (2011).

Table 9 reports the estimated factor prices. Both factor prices for traded and nontraded risk are positive and statistically significant. Quantitatively, one additional unit of a currency strategy's exposure to the U.S. nontraded consumption risk (i.e., β_N increases by one) boosts the expected excess return on the strategy by 29.2 basis points. The positive estimate presents an empirical support to the finding of Proposition 3 that fluctuations in the U.S. nontraded consumption growths are a priced risk in currency strategies from the U.S. investors' perspective. One additional unit of exposure to the U.S. traded consumption risk also boosts the expected excess return on the strategy by 26.8 basis points. This result shows that fluctuations in the U.S. traded consumption are also perceived as a risk by U.S. investors. The fact that estimates of λ_N^{US} and λ_T^{US} having similar order does not lessen the importance of nontraded output growth risks in asset pricing. This is

[25] We do not use U.S. output series (in the output approach to GDP) for the current factor analysis. This is because for traded components, due to trades, the U.S. traded output is not the same as the U.S. traded consumption. And in the theory being tested, it is the consumption risk that matters for the pricing.

[26] We also use lagged values of the carry trade portfolio returns as instruments.

because these risks contribute essentially to *both* factor prices (15), (16). Table 9 also reports the estimated consumption beta's for four currency portfolios. Values of betas vary across portfolios as well as factors, detailing the exposures of different currency strategies to two U.S. consumption risk factors. The exposures $\{\beta_N^P\}$ to the nontraded U.S. output growth risk do not necessarily sort along portfolios $\{P\}$ because the nontraded growth risk enters both traded and nontraded factor prices in equilibrium, per the discussion below (16).

5 Conclusion

This paper examines the effects of national nontraded output growth risk on international asset pricing. Nontraded output growth risk is impactful because this output makes a large share of GDP and is consumed almost entirely by home population. In contrast, country-specific traded output growth risk can be diversified by means of commodity trades. Hence our analysis calls for a careful decomposition of GDP into traded and nontraded output components before assessing its role on the determination of asset prices.

Nontraded output shocks are nevertheless not entirely internalized by host countries because countries engage in international trades in other substitutable goods as well. While, to a certain extent, trades mitigate the impact of home nontraded output risk on the host economy, trades also transmit foreign nontraded output risks home, thus broaden the impact of nontraded output shocks among all trade partners in two-way directions. This mechanism contributes to the pricing of all quantities in international markets, including interest rates, exchange rates, and currency premia. In contrast, as country-specific traded output risks are first pooled into the aggregate traded output, they are diversified fairly across countries, and largely drop from strategies involving off-setting positions in different national markets.

The frameworks in which a risk, apparently intrinsic to only one party, actually affects other parties are pervasive in the real world, e.g., social network settings and intermediary systems. The insights of nontraded risk would provide a pricing perspective to the emerging literature on financial networks. We hope to address these topics in future work.

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Table 1: Services' tradabilities, 1970-2016

Country	Measure	Construction (isic4 F)	Trade; repairs; transp., accom.; food serv. activ. (isic4 G-I)	Infor., commun.; financial; business activ. (isic4 J-N)	Community, Social, Personal serv. (isic4 O-U)
Australia	Tradability (%)	n.a.	0.00	0.01	0.12
	Fraction of GDP (%)	5.56	16.71	16.90	14.94
Czech Republic	Tradability (%)	4.01	15.73	0.63	0.11
	Fraction of GDP (%)	6.80	20.38	19.23	14.14
Denmark	Tradability (%)	2.79	13.05	0.71	0.06
	Fraction of GDP (%)	5.37	16.18	22.21	23.76
Euro	Tradability (%)	0.38	9.51	1.36	0.00
	Fraction of GDP (%)	4.30	16.31	26.04	17.87
Japan	Tradability (%)	n.a.	0.00	0.01	0.03
	Fraction of GDP (%)	5.66	21.02	19.93	14.08
Korea	Tradability (%)	10.47	7.97	0.19	0.07
	Fraction of GDP (%)	7.23	16.44	18.46	17.79
Luxembourg	Tradability (%)	17.18	15.89	0.32	0.09
	Fraction of GDP (%)	5.16	15.08	46.29	13.57
Norway	Tradability (%)	3.14	3.38	0.22	0.03
	Fraction of GDP (%)	4.09	15.47	16.74	19.72
Poland	Tradability (%)	2.17	12.77	0.90	0.06
	Fraction of GDP (%)	6.58	22.11	19.67	14.60
Slovakia	Tradability (%)	4.98	6.93	0.76	0.02
	Fraction of GDP (%)	6.36	19.73	17.55	13.99
Sweden	Tradability (%)	2.95	16.09	0.49	0.05
	Fraction of GDP (%)	4.55	15.23	23.99	19.83
United Kingdom	Tradability (%)	1.53	11.40	1.52	0.77
	Fraction of GDP (%)	7.00	16.56	25.78	17.42
United States	Tradability (%)	0.54	3.25	0.19	0.12
	Fraction of GDP (%)	4.88	14.90	33.28	15.23

Notes: This table lists the mean of country-specific tradabilities and sizes of different service categories for a representative set of 13 OECD countries, 1970-2016. Tradability of services is (one half of) the ratio of total real export and import over total real output of these services by a country (see (19)). Fraction of GDP (or size) of services is the ratio of total real output of these services over the real GDP of a country. See section 4.1 and data appendix for further details.

Table 2: Tradability of major U.S. industries, 1970-2016

	ISIC rev. 4 designation	Industries	US-specific tradability (%)	OECD tradability (%)
1	D13T15	Textiles, wearing apparel, leather and related products	112.2	94.63
2	D46	Wholesale trade, except of motor vehicles and motorcycles	50.69	49.89
3	D26T33	Machinery and equipment; Furniture; other manufacturing	42.39	30.61
4	D05T09	Mining and quarrying	38.32	39.64
5	D24T25	Basic metals and fabricated metal products	27.55	41.11
6	D19T23	Chemical, rubber, plastics, fuel, non-metallic mineral	25.82	42.35
7	D10T12	Food products, beverages and tobacco	22.68	36.53
8	D16T18	Wood and paper products, and printing	17.68	25.13
9	D01T03	Agriculture, hunting, forestry and fishing	17.36	16.91
10	D49T52	Transportation	6.36	2.87
11	D77	Rental and leasing activities	1.59	0.9
12	D64T66	Financial and insurance activities	1.01	0.62
13	D55T56	Accommodation and food service activities	0.78	0.4
14	D41T43	Construction	0.49	0.19
15	D85	Education	0.04	0.02

Notes: This table lists the tradabilities of major U.S. industries, along with their US-specific and OECD tradabilities. The industries are classified by ISIC Revision 4. US-specific tradability is (one half of) the ratio of total real export and import over total real output by the U.S. of the industry (see (19)). OECD tradability for a industry is defined similarly, but with export, import, and output replaced by total-OECD counterparts (see (18)). See section 4.1 and data appendix for further details.

Table 3: Trade-closedness regression

	(1)	(2)	(3)	(4)
Growth variance	-.10071	-.00575	.16469	.27006
	(3.5529)	(3.9556)	(3.8726)	(3.8588)
Closeness	-.0229**	-.02305**	-.02358**	-.02359**
	(.01069)	(.01012)	(.01018)	(.01022)
Variance $\times$ Closeness	-15.928	-16.331***	-16.645***	-16.825***
	(10.061)	(5.38)	(5.2726)	(5.3724)
Growth mean		-.05565	-.05648	-.03103
		(.74611)	(.74804)	(.74901)
GDP size			-.19958	-.18546
			(.13738)	(.14104)
Inflation volatility				.00043*
				(.00024)
Constant	.01605	.01772	.02066	.01945
	(.01385)	(.01276)	(.01309)	(.01319)
Adjusted R^2	0.35395	0.34146	0.32937	0.31859
N	93	93	93	93

Notes: Panel regression $r_t^H = \alpha + \beta_\sigma^H (\sigma_t^H)^2 + \beta_C^H \mathcal{C}_t^H + \beta_{\sigma C}^H (\sigma_t^H)^2 \mathcal{C}_t^H + \beta_x X_t + \epsilon_t^H$ to examine the effects of output volatility σ^H and trade closedness $\mathcal{C}^H$ on interest rate r^H. Panel regression includes country and time fixed effect. Values in parentheses below each regression coefficient are robust standard errors. 10%, 5%, 1% significance levels of two sided t-statistic are indicated by *, ** and ***, respectively.

Dependent variable is the annualized real interest rate, proxied by the short-term Treasury bill rate minus inflation rate, averaged over each period. The sample consists of annual data series for OECD countries 1970-2016. Current members of European Monetary Union are dropped from the sample when they joined the Union, and replaced by a single observation for Eurozone. Growth variance is the annualized variance of growth rate of per-capita real GDP. Closedness is one subtracted by the ratio of country's total real trade over country's real GDP (see (17)). Growth mean is the annualized mean of growth rate of per-capita real GDP. GDP size is the ratio of country's real GDP over total real GDP of OECD member states. Inflation volatility is the standard deviation of country's inflation growth. All mean and variance are computed for each of the non-overlapping periods 1970-1979, 1980-1989, 1990-1999, 2000-2009, 2010-2016. See data appendix for further details.

Table 4: Multi-industry nontradability-dummy regression

	(1)	(2)	(3)	(4)
Growth variance	.0088	.01581**	.01782***	.01884***
	(.00793)	(.00775)	(.00631)	(.00517)
Nontradability dummy	-.01436	-.00881	-.01001	-.01174
	(.00993)	(.01006)	(.00977)	(.00996)
Variance × dummy	4.7e-06	-.00798***	-.00838***	-.00734**
	(.00445)	(.00198)	(.0026)	(.0035)
Growth mean		.01923***	.01784***	.01495**
		(.00683)	(.00673)	(.00606)
GDP size			-.32274	-.34729
			(.35161)	(.33836)
Inflation volatility				-.00092**
				(.00045)
Constant	.03504***	.03492***	.03864***	.03924***
	(.00831)	(.00828)	(.00733)	(.00731)
Adjusted R^2	0.52077	0.52209	0.52781	0.54151
N	1475	1475	1404	1404

Notes: Panel regression $r^H_{i,t} = \alpha + \beta_\sigma(\sigma^H_{i,t})^2 + \beta_d d_{i,t} + \beta_{\sigma d}(\sigma^H_{i,t})^2 d_{i,t} + \beta_x X^H_{i,t} + \epsilon^H_{i,t}$ to examine the effects of industry-level output volatility σ^H_i and its dummy nontradability d_i on interest rate r^H. Panel regression includes country and industry fixed effect. Values in parentheses below each regression coefficient are robust standard errors. 10%, 5%, 1% significance levels of two sided t-statistic are indicated by *, ** and ***, respectively.

Dependent variable is the annualized real interest rate, proxied by the short-term Treasury bill rate minus inflation rate, averaged over each period. The sample consists of annual data series for OECD countries 1970-2016. Current members of European Monetary Union are dropped from the sample when they joined the Union, and replaced by a single observation for Eurozone. Growth variance is the annualized variance of growth rate of per-capita country-specific industries' real output. Nontradability dummies are at industry level; they assume value 1 for industries classified as nontraded sectors (Construction; Wholesale and retail trade; Repair of motor vehicles and motorcycles; Transportation and storage; Accommodation and food service activities; Information and communication; Financial and insurance activities; Real estate, renting and business activities; Community, social and personal services), and 0 otherwise. Growth mean is the annualized mean of growth rate of per-capita country-specific industries' real output over the corresponding period. GDP size is the ratio of country's real GDP over total real GDP of OECD member states. Inflation volatility is the standard deviation of country's inflation growth. All mean and variance are computed for each of the non-overlapping periods 1970-1979, 1980-1989, 1990-1999, 2000-2009, 2010-2016. See data appendix for further details.

Table 5: Multi-industry global nontradability regression

	(1)	(2)	(3)	(4)
Growth variance	.06069	.06354**	.06178**	.06037**
	(.04072)	(.02689)	(.02275)	(.02279)
Global nontradability	.00142	.00136	.00171	.00182
	(.002)	(.00211)	(.00209)	(.0021)
Variance × nontradability	-.05143	-.05457**	-.04947**	-.04851**
	(.03973)	(.02547)	(.02158)	(.02172)
Growth mean		.00219	-.00373	-.00439
		(.01375)	(.01274)	(.01276)
GDP size			-.47794*	-.4621*
			(.25782)	(.24861)
Inflation volatility				.00036
				(.00021)
Constant	.01098	.01097	.03733*	.03602*
	(.00782)	(.00783)	(.01897)	(.0188)
Adjusted R^2	0.60131	0.60092	0.60974	0.61546
N	944	944	909	909

Notes: Panel regression $r_{i,t}^H = \alpha + \beta_\sigma (\sigma_{i,t}^H)^2 + \beta_\tau \tau_{i,t} + \beta_{\sigma\tau} (\sigma_{i,t}^H)^2 \tau_{i,t} + \beta_x X_{i,t}^H + \epsilon_{i,t}^H$ to examine the effects of industry-level output volatility σ_i^H and its global nontradability τ_i on interest rate r^H. Panel regression includes industry as well as time fixed effect. Values in parentheses below each regression coefficient are robust standard errors. 10%, 5%, 1% significance levels of two sided t-statistic are indicated by *, ** and ***, respectively.

Dependent variable is the annualized real interest rate, proxied by the short-term Treasury bill rate minus inflation rate, averaged over each period. The sample consists of annual data series for OECD countries 1970-2016. Current members of European Monetary Union are dropped from the sample when they joined the Union, and replaced by a single observation for Eurozone. Growth variance is the annualized variance of growth rate of per-capita country-specific industries' real output. Here nontradability is a global measure and at industry level; it is one subtracted by one half of the ratio of global total real trade (i.e., import plus export) in an industry over the global total real output in that industry (see (18)). Growth mean is the annualized mean of growth rate of per-capita country-specific industries' real output. GDP size is the ratio of country's real GDP over total real GDP of OECD member states. Inflation volatility is the standard deviation of country's inflation growth. All mean and variance are computed for each of the non-overlapping periods 1970-1979, 1980-1989, 1990-1999, 2000-2009, 2010-2016. See data appendix for further details.

Table 6: Multi-industry country-specific nontradability regression

	(1)	(2)	(3)	(4)
Growth variance	.03111	-.02553	-.02175	-.02229
	(.03801)	(.03841)	(.03651)	(.0368)
Industry nontradability	-2.4e-05***	-2.4e-05***	-2.4e-05***	-2.4e-05***
	(3.7e-06)	(3.8e-06)	(3.9e-06)	(3.9e-06)
Variance × nontradability	-.00013***	-9.4e-05***	-9.3e-05***	-9.3e-05***
	(2.5e-05)	(2.9e-05)	(2.7e-05)	(2.7e-05)
Growth mean		-.05134	-.05272	-.05296
		(.0379)	(.03634)	(.03641)
GDP size			-.32268	-.30604
			(.27301)	(.25927)
Inflation volatility				.0002
				(.00054)
Constant	.00658	.00854	.02732	.02601
	(.0055)	(.0057)	(.01976)	(.01913)
Adjusted R^2	0.62277	0.62757	0.63225	0.63223
N	585	585	556	556

Notes: Panel regression $r_{i,t}^H = \alpha + \beta_\sigma(\sigma_{i,t}^H)^2 + \beta_\tau \tau_{i,t}^H + \beta_{\sigma\tau}(\sigma_{i,t}^H)^2 \tau_{i,t}^H + \beta_x X_{i,t}^H + \epsilon_{i,t}^H$ to examine the effects of industry-level output volatility σ_i^H and its country-specific nontradability τ_i^H on interest rate r^H. Panel regression includes country, industry as well as time fixed effect. Values in parentheses below each regression coefficient are robust standard errors. 10%, 5%, 1% significance levels of two sided t-statistic are indicated by *, ** and ***, respectively.

Dependent variable is the annualized real interest rate, proxied by the short-term Treasury bill rate minus inflation rate, averaged over each period. The sample consists of annual data series for OECD countries 1970-2016. Current members of European Monetary Union are dropped from the sample when they joined the Union, and replaced by a single observation for Eurozone. Growth variance is the annualized variance of growth rate of per-capita country-specific industries' real output. Nontradability is a country-specific measure and at industry level; it is one subtracted by one half of the ratio of country's real trade (i.e., import plus export) in an industry over the country's real output in that industry (see (19)). Growth mean is the annualized mean of growth rate of per-capita country-specific industries' real output. GDP size is the ratio of country's real GDP over total real GDP of OECD member states. Inflation volatility is the standard deviation of country's inflation growth. All mean and variance are computed for each of the non-overlapping periods 1970-1979, 1980-1989, 1990-1999, 2000-2009, 2010-2016. See data appendix for further details.

Table 7: Currency portfolios (one-dimensional sorting)

	Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4	Long-short (1)-(4)
Sorted by NT x Size					
Return (%)	2.159	0.685	-0.520	-1.067	3.226
Sharpe ratio (%)	23.450	7.257	-5.466	-13.253	44.763
Sorted by NT					
	Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4	Long-short (1)-(4)
Return (%)	1.243	0.680	1.305	-0.014	2.647
Sharpe ratio (%)	13.651	9.357	13.611	-14.571	36.492
Sorted by Size					
	Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4	Long-short (1)-(4)
Return (%)	1.030	0.276	-2.657	0.418	0.611
Sharpe ratio (%)	11.056	2.6448	-22.143	6.358	8.997

This figure presents (annualized) means and Sharpe ratios of real excess returns on four sorted quarterly rebalanced currency portfolios. The sample consists of quarterly data series for period 1976-2016. The portfolio are constructed by sorting currencies into four groups at beginning of quarter t based on the value of (1) nontraded real output growth variance $\times$ real GDP's relative size, (2) nontraded real output growth variance, and (3) real GDP's relative size, over the previous 15 quarters. Portfolio 1 contains currencies with the lowest sorted value, portfolio 4 the highest. Due to unbalances in spot and forward exchange rate series, countries' data become available at different times, and number of countries changes over time. See data appendix for further details.

Table 8: Currency portfolios (two-dimensional sorting)

Panel A. Return (%)			
	Small Size	Large Size	Long-short based on Size
Low NT	1.097	1.056	0.041
High NT	0.780	-3.036	3.816
Long-short based on NT	0.318	4.093	

Panel B. Sharpe ratio (%)			
	Small Size	Large Size	Long-short based on Size
Low NT	12.067	15.498	0.651
High NT	8.270	-24.575	32.667
Long-short based on NT	4.205	37.247	

This figure presents (annualized) means and Sharpe ratios of real excess returns on four quarterly rebalanced currency portfolios, as well as the resulting long-short portfolios to U.S. investors. The sample consists of quarterly data series for period 1976-2016. The sorted portfolios are constructed by sorting currencies into four groups at beginning of quarter t based on 2 dimensions, the real GDP's relative size and the value of nontraded real output growth variance over the previous 15 quarters. The long-short portfolios are constructed by either (i) keeping the same level (small/large) of real GDP relative size and long-short portfolios based on the value of nontraded real output growth variance, or (ii) keeping the same level (high/low) of nontraded growth variance and long-short portfolios based on GDP relative size. Due to unbalances in spot and forward exchange rate series, countries' data become available at different times, and number of countries changes over time. See data appendix for further details.

Table 9: Estimation of factor prices in linear factor models

		Nontraded consumption	Traded consumption
Factor prices (%)		0.292***	0.268**
		(0.0035)	(0.132)
beta's	port. 1	-0.716	0.266
	port. 2	0.405	-0.117
	port. 3	-1.287	0.200
	port. 4	0.543	-0.297

Note: Upper panel reports the GMM annualized estimates of the factor prices (in percentage points), lower panel reports the estimates of the portfolios' exposures to risk factors (i.e. beta's) in the carry trade linear factor model using four quarterly rebalanced currency portfolios as test assets. HAC standard errors for the factor prices are obtained by two-stage GMM procedure using constant and lagged carry trade portfolio returns as instruments, and are reported in parenthesis. The currencies are sorted based on the product of nontraded real output growth variance and real GDP's relative size. The sample consists of quarterly data series for the period 1976-2016.

Figure 1: Nontraded real output-over-GDP ratio, averaged over 1970-2016, for OECD countries

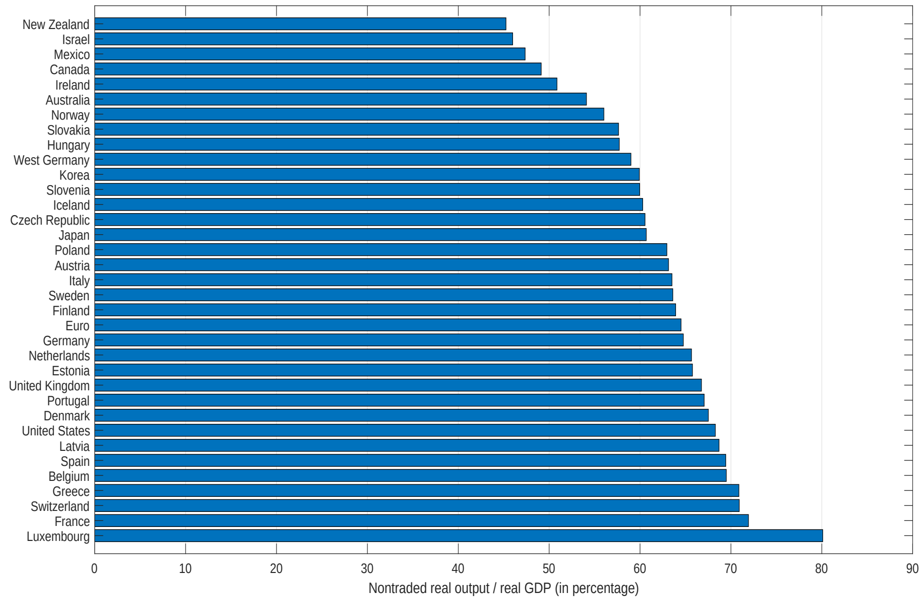


Figure 2: Nontraded output risk versus Size, averaged over 1970-2016, for OECD countries

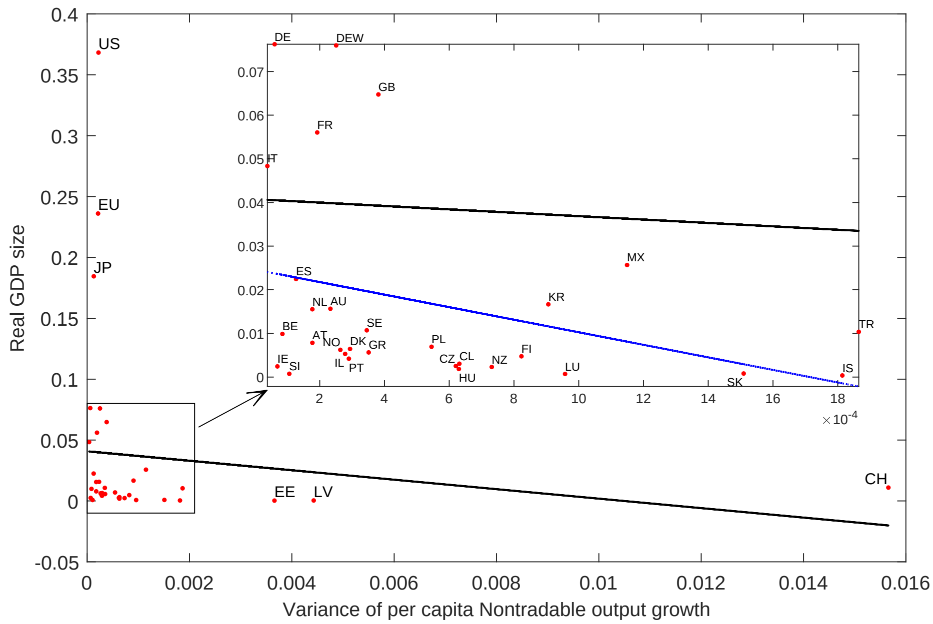


Figure 3: The combined effect of NT output risk and Size, averaged over 1970-2016, for OECD countries

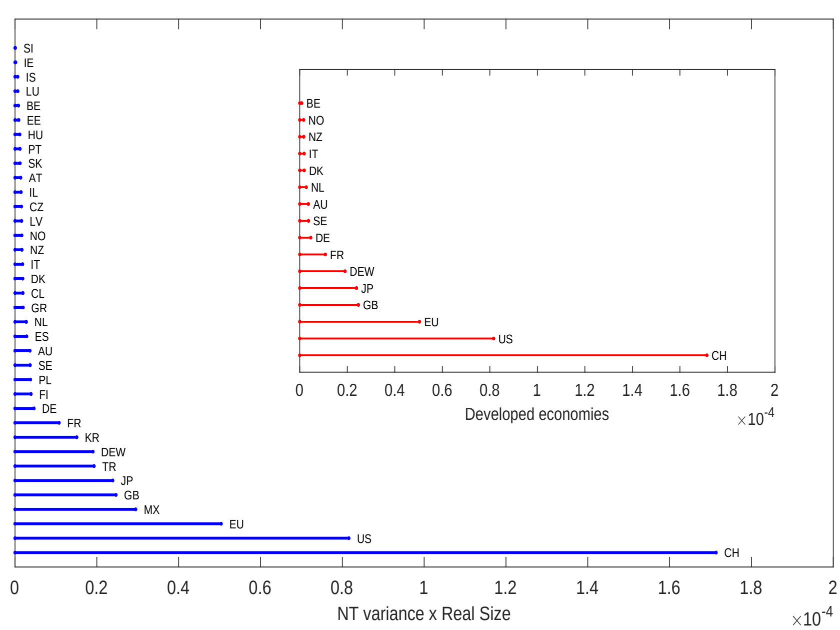


Figure 4: Volatility of per-capita nontraded real output growth, averaged over 1970-2016, for OECD countries

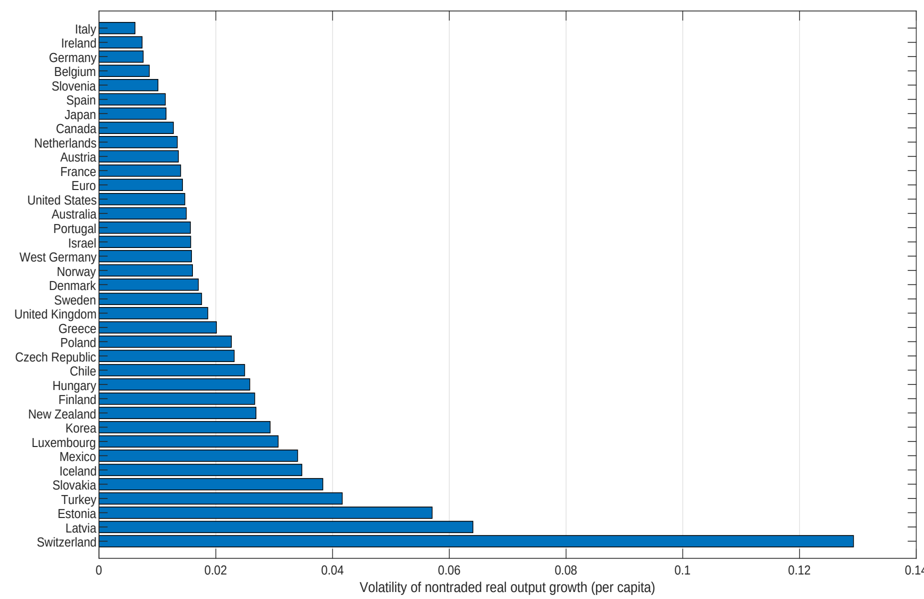


Figure 5: Real Trade-to-GDP ratio (i.e., openness), averaged over 1970-2016, for OECD countries. Trade is defined as the sum of export and import of the country.

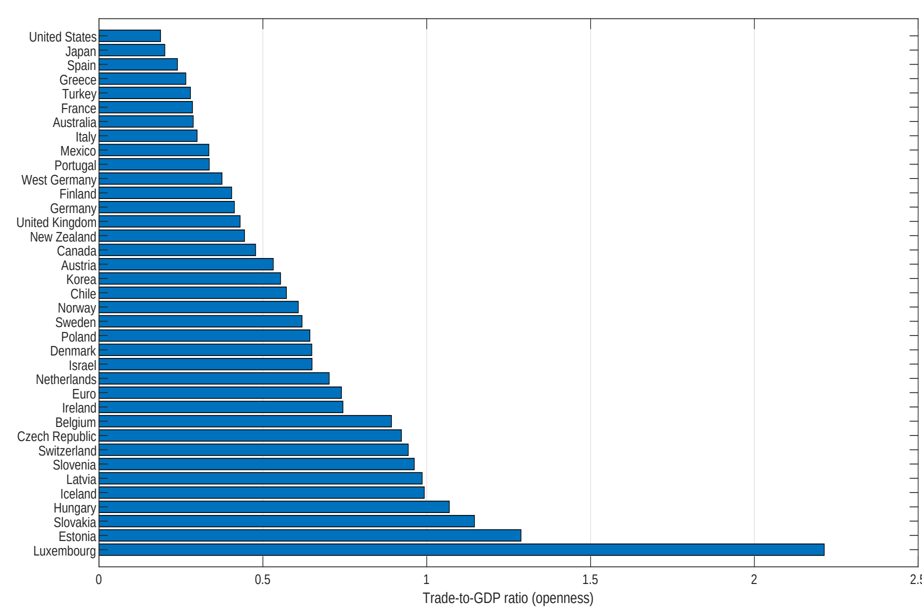
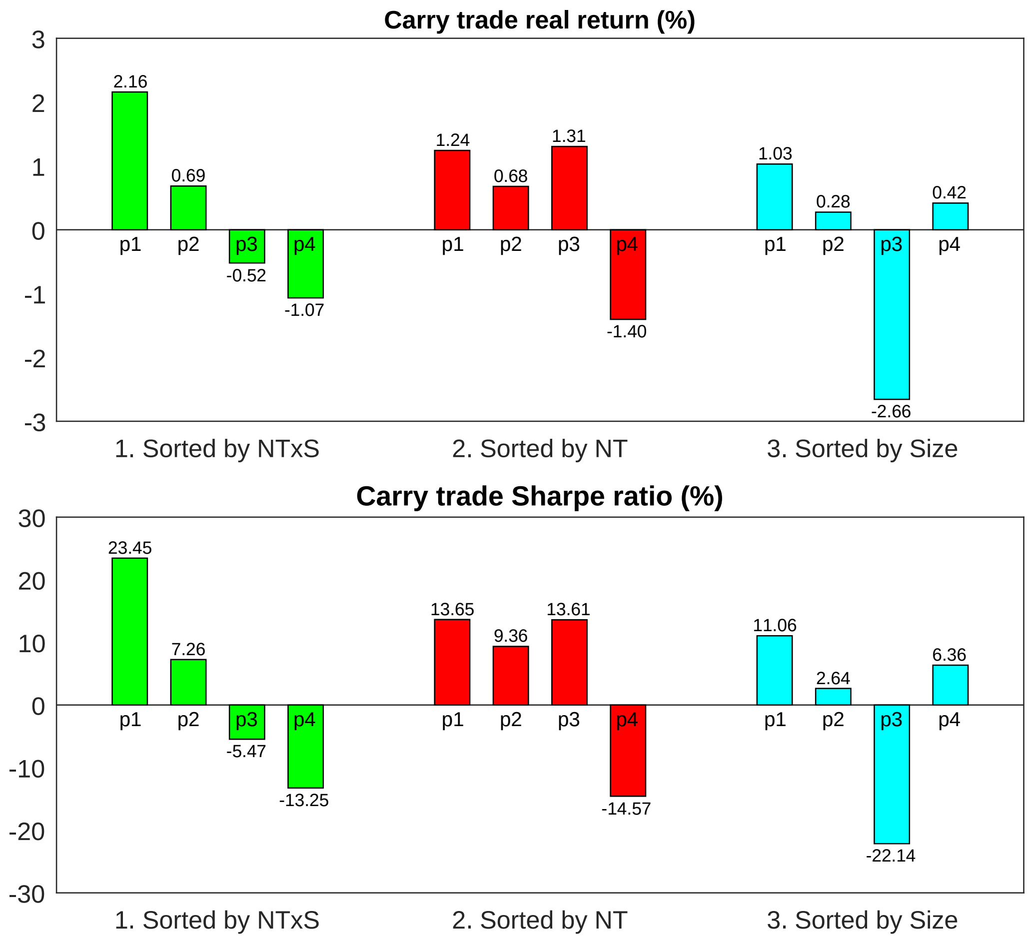
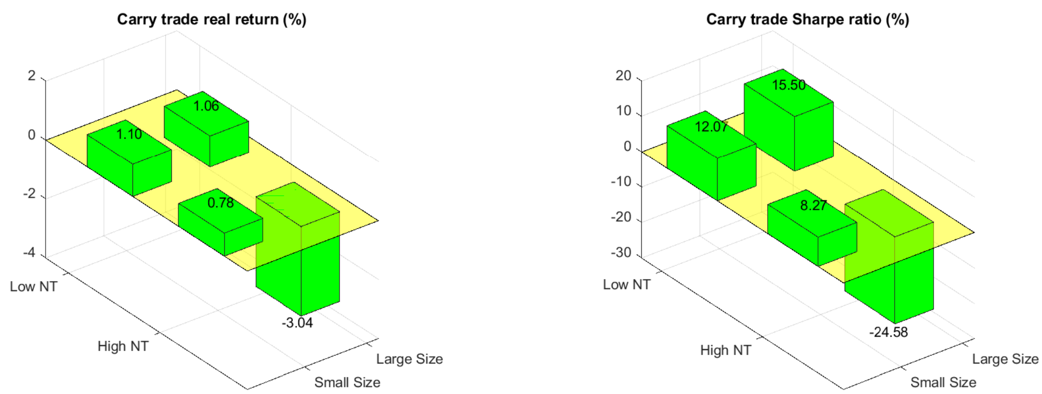


Figure 6: Carry trade real excess returns and Sharpe ratios for portfolios sorted on nontraded output risk and size



This figure presents (annualized) means and Sharpe ratios of real excess returns on four quarterly rebalanced currency portfolios to U.S. investors. The sample consists of quarterly data series for period 1976-2016. The portfolio are constructed by sorting currencies into four groups at beginning of quarter t based on (1) the value of nontraded real output growth variance $\times$ real GDP's size (2) the value of nontraded real output growth variance, and (3) real GDP's size. Variance is calculated over the previous 15 quarters. Portfolio 1 contains currencies with the lowest sorted value, portfolio 4 the highest. Due to unbalances in spot and forward exchange rate series, countries' data become available at different times, and number of countries changes over time. See data appendix for further details.

Figure 7: Two-dimensional Sort



This figure presents (annualized) means and Sharpe ratios of real excess returns on four quarterly rebalanced currency portfolios to U.S. investors. The sample consists of quarterly data series for period 1976-2016. The portfolios are constructed by sorting currencies into four groups at beginning of quarter t based on 2 dimensions, the real GDP's size and the value of nontraded real output growth variance over the previous 15 quarters. Due to unbalances in spot and forward exchange rate series, countries' data become available at different times, and number of countries changes over time. See data appendix for further details.

Appendices

A Data Sources

The empirical part of the current paper concerns only countries that belong to the Organisation for Economic Co-operation and Development (OECD) principally because we reasonably expect that data quality for these developed economies should be higher than the rest of the world.

OECD countries: currently, there are 36 OECD member states listed in the following table (along with their country code).

Country	Code	Country	Code	Country	Code
Australia	AU	Greece	GR	New Zealand	NZ
Austria	AT	Hungary	HU	Norway	NO
Belgium	BE	Iceland	IS	Poland	PL
Canada	CA	Ireland	IE	Portugal	PT
Chile	CL	Israel	IL	Slovakia	SK
Czech Republic	CZ	Italy	IT	Slovenia	SI
Denmark	DK	Japan	JP	Spain	ES
Estonia	EE	Korea	KR	Sweden	SE
Euro	EU	Latvia	LV	Switzerland	CH
Finland	FI	Luxembourg	LU	Turkey	TR
France	FR	Mexico	MX	United Kingdom	GB
Germany	DE	Netherlandss	NL	United States	US
West Germany	DEW				

Eurozone countries: among OECD states, the following 15 belong to the Economic and Monetary Union (a.k.a., Eurozone or Euro area) with respective adopting date in the parenthesis; Austria (01/01/1999), Belgium (01/01/1999), Estonia (01/01/2011), Finland (01/01/1999), France (01/01/1999), Germany (01/01/1999), Greece (01/01/2001), Ireland (01/01/1999), Italy (01/01/1999), Luxembourg (01/01/1999), Netherlandss (01/01/1999), Portugal (01/01/1999), Slovak Republic (01/01/2009), Slovenia (01/01/2007), Spain (01/01/1999). (Only two other Eurozone states are Cyprus and Malta, but they do not belong to OECD and are not considered in the empirical analysis of the current paper.)

OECD "Aggregate National Accounts" (ANA) database, downloaded from stat.oecd.org, contains the following information used in our annual interest rate regression:

- "Gross domestic product" contains annual outputs as well as expenditures in national currency, and corresponding deflators. Data is available for the total gross domestic product (B1-GA), as well as for individual service categories, i.e., Construction (ISIC Rev.4 B1GVF); Distributional trade, repairs, transportation, accommodation, food service activities (B1GVG_I); Information and communication (B1GVJ); Financial and insurance activities (B1GVK); Real estate activities (B1GVL); Professional, scientific, technical activities, administrative, support service activities (B1GVM_N); Public administration, compulsory services, education, human health (B1GVO_Q); and Other service activities (B1GVR_U).
- "Export and import": export and import for the total economy, in local currency, as well as their deflators;
- "Population";
- "Exchange rate" data;

OECD "Quarterly National Accounts" (QNA) database, downloaded from stat.oecd.org, contains the following information used in our carry trade analysis:

- "Gross domestic product" contains quarterly outputs and expenditure in national currency, and corresponding deflators. The classification is the same as in the ANA database.
- Population: Turkey and Korea do not have quarterly data in the QNA, we use the annual data from ANA instead, and intrapolate for quarterly points. For other countries, not all countries have data starting from 1970. For those cases, similarly, we supplement by annual population data and intrapolate to fill in the missing points.

OECD "Key Short-term Economics Indicators" (Key STEI) database, downloaded from stat.oecd.org, provides data on consumer price index. This series is available for both annual and quarterly frequency. Inflation is then computed as the percentage change of the price index. Inflation volatility is calculated as the standard deviation of inflation growth.

"Treasury bill rates" is downloaded from Thomson Reuters Datastream. The rate is 3-month interest rates at quarterly frequency. Not every countries have interest rate available from 1970:

Austria series starts in Q3 1999, Chile in Q3 2011, Czech Republic in Q4 2002, Denmark in Q3 1988, Estonia in Q1 2011, Finland in Q1 1987, France in Q2 1989, Ireland in Q1 1984, Israel in Q1 1995, Italy in Q2 1988, Japan in Q2 1993, Korea in Q4 2004, Latvia in Q3 1994, Luxembourg in Q1 2003, Mexico and Netherlands in Q1 1978, New Zealand in Q2 1985, Norway in Q2 2003, Poland in Q3 1993, Portugal in Q2 1993, Slovakia in Q3 1994, Slovenia in Q1 1999, Spain in Q2 1988, Sweden in Q3 1989, Switzerland in Q2 1974, Turkey in Q3 2006, United Kingdom in Q2 1985, United States in Q1 1972.

"Spot and 3-month forward exchange rates" is downloaded from Datastream. These data are provided by Barclays Capital and WM/Reuters (WMR). In cases where data for one currency is available from both sources, the longer series is used. Exchange rates data quoted against the USD is available from 1983. Exchange rate data quoted against the Pound is available from 1976. We use the Pound/USD exchange rate to convert these exchange rates against Pound to exchange rates against USD for the earlier period.

"Quarterly U.S. consumption expenditures and price indexes" are series from U.S. Bureau of Economic Analysis. Table 2.3.5. therein contains "Personal Consumption Expenditures by Major Type of Product". Table 2.3.4. contains "Price Indexes for Personal Consumption Expenditures by Major Type of Product". We identify the personal consumption expenditures on services (i.e., the component "Services" listed in these tables) as the U.S. nontraded consumption. We identify the personal consumption expenditures on other goods (i.e., the component "Goods" listed in these tables) as the U.S. traded consumption. These quarterly series start well before 1970 (all our empirical studies in the current paper concern periods starting in 1970 or later).

OECD "Trade by Enterprise Characteristics" (TEC) database contains international annual trade in goods data broken down by different categories of enterprises. This data not only gives total import and export data, but also gives extra-EU trade data, facilitating the calculation of trade data for the EU zone. TEC data gives import and export for different types of goods (from ISIC Rev.4 code 1 to 99, as well as from code A to S). However, data is only available from 2007. We will combine with with other import and export data available in STAN3 database to have a full historical time series of trade data in various classifications of goods and services.

OECD "Structural Analysis" (STAN) database provides, for each OECD country, the annual nominal output series (in national currency) and the corresponding deflator series for various industries. The ISIC Rev.4 revision (STAN4 - we use the latest data available at the starting of the research project time, i.e., November 2017 edition) has 30 countries, and uses 2010 base year for the deflators. The ISIC Rev.3 revision (STAN3 - edition 2011) has 33 countries, and uses 2000 base year for the deflators. However, STAN3 also provides country-specific annual nominal import and export series (in national currency) and the corresponding deflator series. Therefore we combine STAN4, STAN3 and TEC to obtain a full data set of real outputs as well as real import/export for various industries.

B Combine STAN3, STAN4 and TEC Data

We need to merge the TEC, STAN 4 (which use ISIC Rev.4 classification) and STAN3 (which uses ISIC Rev.3 classification) to create a full data set of real outputs and trade for different good/industry classification. Information on International Standard Industrial Classification of All Economic Activities (ISIC), different classification revisions, as well as link tables between them, can be found in https://unstats.un.org/unsd/classifications/Family/Detail/27.

It should be noted that the codes mapping between ISIC Rev.3 and ISIC Rev.4 is not a straightforward one-to-one mapping. One code in Rev.3 can be broken into different components of various codes in Rev.4. Table 10 shows the industries that we can have reasonably accurate matching that we use in our industry analysis.

C Derivations and Proofs

This appendix presents technical derivations of all results in the main text, concerning the basic model. In this basic model with complete market and no trade friction (Section 2), the equilibrium marginal utilities of traded consumption equal across countries, which give K FOCs; $M_T = \frac{\partial U^H}{\partial C_T^H}$ $\forall H = 1, \ldots, K$. The market clearing condition for traded good presents another equation to solve for $K+1$ unknowns; $\{C_T^H\}_{H=1}^K$ and M_T. We log-linearize the system to obtain an approximative solution in closed form.

Equilibrium log consumption (3): Plugging the expression (1) for U^H into the FOC (2), and log-linearizing this FOC around the steady state corresponding to the symmetric configuration

Table 10: Matching between ISIC Rev.3 and ISIC Rev.4

ISIC Rev.4 codes	ISIC Rev.4 industry names	Matching ISIC Rev.3 codes
D01T03	Agriculture, hunting, forestry and fishing	01-05
D05T09	Mining and quarrying	10-14
D10T12	Food products, beverages and tobacco	15-16
D13T15	Textiles, wearing apparel, leather and related products	17-19
D16T18	Wood and paper products, and printing	20; and 21-22
D19T23	Chemical, rubber, plastics, fuel products and other non-metallic mineral products	23-25; and 26
D24T25	Basic metals and fabricated metal products, except machinery and equipment	27-28
D26T28 + D29T30 + D31T33	Machinery and equipment; Transport equipment; Furniture; other manufacturing; repair and installation of machinery and equipment	29-33; and 34-35; and 36
D35+D36	Electricity, gas, steam and air conditioning supply Water collection, treatment and supply	40-41
D41T43	Construction	45
D46	Wholesale trade, except of motor vehicles and motorcycles	51
D49 + D50 + D51 + D52	Land transport and transport via pipelines; Water transport; Air transport; Warehousing and support activities for transportation	60-63
D53 + D61	Postal and courier activities; Telecommunications	64
D55T56	Accommodation and food service activities	55
D64T66	Financial and insurance activities	65-67
D77	Rental and leasing activities	71
D85	Education	80
D86T88	Human health and social work activities	85
D94	Activities of membership organizations	91
D96	Other personal service activities	93
D97T98	Activities of households as employers; undifferentiated activities of households for own use	95
D99	Activities of extraterritorial organizations and bodies	99

Note: This table gives the industry codes we use in our industry analysis, for those industries that we can match closely between ISIC Rev.4 and ISIC Rev.3 codes. For other codes, one code in Rev.4 matches to different components of various codes in Rev.3 of so no accurate matching can be done.

$\{\delta_T^H = \delta_N^H;\ \delta_T^H/\Lambda^H = \delta_T^F/\Lambda^F\}$ yield an approximate equation[27]

$$\begin{aligned} m_T &\approx \lambda^H - \rho t + (\epsilon - \gamma)(\omega_T c_T^H + \omega_N \delta_N^H) - \epsilon c_T^H + \log \omega_T \\ &= \lambda^F - \rho t + (\epsilon - \gamma)(\omega_T c_T^F + \omega_N \delta_N^F) - \epsilon c_T^F + \log \omega_T. \end{aligned}$$

Similarly, log-linearizing the traded good market clearing equation yields (where $\lambda = \log \Lambda = \log \sum_H^K \Lambda^H$)

$$\sum_H^K \frac{\Lambda^H}{\Lambda} c_T^H = \delta_T + \sum_H^K \frac{\Lambda^H}{\Lambda} \lambda^H - \lambda. \tag{22}$$

Substituting c_T^H from the first equation above into the second equation gives m_T, and then c_T^H in (3).

Country-specific stochastic discount factor (4): In pricing country-specific financial assets, the appropriate measures are country-specific consumption baskets (i.e., national currencies in the current consumption-based setting). A country-specific consumption basket is the lowest-cost bundle of traded and nontraded consumption that delivers a unit of country's utility, given the consumption goods' prices $\{P_T^H \equiv 1, P_N^H\}$ (in term of traded goods). The basket's composition $\{C_T^H, C_N^H\}$ and value P^H solve $\min_{C_T^H, C_N^H} P^H \equiv C_T^H + C_N^H P_N^H$ subject to $\left[\omega_T (C_T^H)^{1-\epsilon} + \omega_N (C_N^H)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} = 1$. Then follows the value of consumption basket in term of traded good

$$P_t^H = \left[\omega_T^{\frac{1}{\epsilon}} + \omega_N^{\frac{1}{\epsilon}} (P_N^H)^{\frac{1-\epsilon}{-\epsilon}}\right]^{\frac{-\epsilon}{1-\epsilon}};$$

From this and M_T above follows the identity in equilibrium $M_t P_t^H = M_t^H$, where $M_t^H \equiv \frac{\partial U^H}{\partial C^H} = e^{-\rho t}(C^H)^{-\gamma}$ and C^H is the country-specific consumption aggregator.[28] The current price of the country-specific risk-free bond (that pays one unit of country-specific consumption basket at time s) is

$$B_{t,s}^H = \frac{1}{P_t^H} E_t\left[\frac{M_s}{M_t} P_s^H\right] = E_t\left[\frac{M_s^H}{M_t^H}\right].$$

It is this pricing equation that establishes the above M_t^H as the country-specific SDF of country H. That is, prices computed using this SDF are in unit of country-specific consumption basket. Note that $m^H = \log M^H = -\rho t - \gamma \log C^H$. Log-linearizing the consumption aggregator C^H and using log equilibrium traded consumption c_T^H in (3) yield country-specific log SDF (4).

[27] Lower-case letters denote logarithms, e.g., $m \equiv \log M$, $\lambda \equiv \log \Lambda$, $c = \log C$, $\delta = \log \Delta$.

[28] In contrast with the country-specific M^H, M_T is the marginal utility with respect to traded good and is same for all countries in complete market settings.

Costly trades: Suppose that home country is an importer (case 1) and trades take place, the variation of social planner's Lagrangian with respect to non-binding consumptions $\frac{\partial}{\partial C_F^H}$, $\frac{\partial}{\partial C_F^F}$ produces FOC $\left(C_H^H + C_F^H\right)^{-\gamma} = (1+\theta)\left(C_H^F + C_F^F\right)^{-\gamma}$. Combining this with binding consumption $C_H^H = \Delta^H$, $C_F^H = 0$, and market clearing condition $C_F^F + (1+\theta)C_F^H = \Delta^F$ yields (7). From this we can also find home SDP $M^H = e^{-\rho t}\left(\Delta^H + C_F^H\right)^{-\gamma}$. The risk-free rate r^H is the opposite to expected growth rate of M^H; $r^H = -\frac{1}{dt}E_t\left[\frac{dM^H}{M^H}\right]$. Plugging equilibrium consumption solutions (7) into M^H, and an application of Ito lemma yields (assuming independent endowments Δ^H, Δ^F)

$$r^H = \rho + \gamma\frac{(1+\theta)\mu^H\Delta^H + \mu^F\Delta^F}{(1+\theta)\Delta^H + \Delta^F} - \frac{1}{2}\gamma(\gamma+1)\frac{(1+\theta)^2(\sigma^H)^2(\Delta^H)^2 + (\sigma^F)^2(\Delta^F)^2}{[(1+\theta)\Delta^H + \Delta^F]^2}$$

which is a more explicit version of (8).

Proof 1 (Proof of Proposition 1) *From (4) follow the partial derivatives* $\frac{\partial m^H}{\partial \delta_N^H} = -\gamma\omega_N\left[1 - \alpha(\gamma-\epsilon)\omega_T\left(1 - \frac{\Lambda^H}{\Lambda}\right)\right]$ *and* $\frac{\partial m^F}{\partial \delta_N^H} = -\gamma\omega_N\left[\alpha(\gamma-\epsilon)\omega_T\frac{\Lambda^H}{\Lambda}\right]$. *Evidently,* $\left|\frac{\partial m^H}{\partial \delta_N^H}\right| > \left|\frac{\partial m^F}{\partial \delta_N^H}\right|$ *because* $\gamma - \epsilon > 0$ *(Assumption 1, Section 2).*

Proof 2 (Proof of Equation (9) and Proposition 2) *We start with the differential representation for SDF* M^H

$$\frac{dM^H}{M^H} = -r^H dt - \eta^H dZ^H; \quad m^H = \log m^H \Longrightarrow dm^H = -\left(r^H + \frac{1}{2}(\eta^H)^2\right) - \eta^H dZ^H.$$

where η^H *is the home market price of risk. Similar relations hold for* M^F *and* m^F. *Plugging these into the realized carry trade excess return* $XR_{t+dt}^{-H,+F}$ *(upper equation in (9)), applying Itô's lemma and taking the conditional expectation yield*

$$\begin{aligned}
E_t\left[XR_{t+dt}^{-H,+F}\right] &= E_t\left[\frac{1+\frac{dM^F}{M^F}}{1+\frac{dM^H}{M^H}}(1+r^F dt) - (1+r^H dt)\right] \\
&= E_t\left[\left(1 + dm^F + \frac{1}{2}(dm^F)^2\right)\left(1 - dm^H + \frac{1}{2}(dm^H)^2\right)(1+r^F dt) - (1+r^H dt)\right] \\
&= E_t\left[dm^F + \frac{1}{2}(dm^F)^2 - dm^H + \frac{1}{2}(dm^H)^2 - dm^H dm^F + r^F dt - r^H dt\right] \\
&= (\eta^H)^2 - \eta^H\eta^F = -Cov_t\left[dm^H, dm^F - dm^H\right],
\end{aligned}$$

which is (9). Next, combining (4) and (10) implies the key expression for expected carry trade excess return (11) of Proposition 2.

Proof 3 (Proof of Proposition 3) *We first develop (14) to obtain more explicit expressions for λ_T and λ_N*

$$\lambda_T^H = Var(f_T^H)b_T + Cov(f_T^H, f_N^H)b_N; \qquad \lambda_N^H = Cov(f_T^H, f_N^H)b_T + Var(f_N^H)b_N.$$

Plugging $\{b_T, b_N\}$ and $\{f_T^H, f_N^H\}$ from (13) into above expressions yields (15) of Proposition 3 and (16) for factor prices associated with nontraded and traded consumption growth risk respectively.

D Online Appendix: Beyond Benchmark Model

The key intuition, developed alongside the basic setting of international finance in the main text, is that the country-specific traded output risk should have a smaller impact on asset prices than the country-specific nontraded output risk because of the diversification in the traded good market. However, the basic model possesses several simplifications, including (i) homogeneous consumption taste for a single common traded good and (ii) complete financial markets worldwide. In this section, we relax these assumptions, verify, and thus strengthen the above intuition to a more realistic and robust economic setting.

D.1 Arbitrary Trade Configuration

Generalized setup: In the current general setting, there are l varieties of traded goods and K types of nontraded goods, and each of the latter is consumed by one respective country. A particular type h of traded goods can be consumed only by some K_h countries. Similarly, a particular country H trades and consumes only some l^H varieties of traded goods. These features aim to capture the realistic and vastly different trade configurations among countries, as well as the vastly different popularity of different traded goods. Moreover, countries can also have country-specific tastes for the traded goods ($\{\omega_h^H\}$) and nontraded good (ω_N^H) that they consume, subject to the conventional normalization $\omega_N^H + \sum_h^{l^H} \omega_h^H = 1$. We also assume that the financial market is complete because contingent claims on all outputs and countries' risk-free bonds are available investment instruments. Consequently, the world's static optimization problem can be used to study the equilibrium behaviors of consumption allocations and asset prices in this economy.

$$\max_{\{C_{h,T}^H\}} \sum_{H=1}^{K} \Lambda^H \frac{e^{-\rho t}}{1-\gamma} \left[\sum_h^{l^H} \omega_{h,T}^H (C_{h,T}^H)^{1-\epsilon} + \omega_N^H (\Delta_N^H)^{1-\epsilon} \right]^{\frac{1-\gamma}{1-\epsilon}} \quad \text{s.t.} \quad \sum_H^{K_h} C_{h,T}^H = \Delta_{h,T} \quad \forall h = 1, \dots, l.$$

Although a country may have different tastes for different goods that they consume, the substitutability between any two varieties, either traded or nontraded, is characterized by the same elasticity coefficient ϵ. It is apparent from the market market clearing conditions that only the aggregate outputs for traded good varieties directly enter the dynamic of the economy. However, the associated output shocks will have different impacts on different countries, depending on their country-specific trade configurations. The current complex setting calls for a quantitative analysis to shed light on the role of these shocks on consumption allocations and prices.

Equilibrium allocations: Combining log-linearization and iteration techniques yield the equilibrium log consumption c_h^H of traded good h by country H

$$
\begin{aligned}
c_h^H &= \delta_{h,T} - (\gamma-\epsilon)\alpha^H \omega_N^H \delta_N^H + (\gamma-\epsilon)\sum_J^{K_h} \alpha^J \frac{\Lambda^J}{\Lambda_h}\left(\omega_N^J \delta_N^J + \sum_j^{l^J} \omega_{j,T}^J \delta_{j,T}\right) \\
&- (\gamma-\epsilon)\alpha^H \sum_i^{l^H} \omega_{i,T}^H \left[\delta_{i,T} + (\gamma-\epsilon)\sum_I^{K_i} \alpha^I \frac{\Lambda^I}{\Lambda_i}\left(\omega_N^I \delta_N^I + \sum_k^{k^I} \omega_{k,T}^I \delta_{k,T}\right)\right], \qquad (23)
\end{aligned}
$$

where in the current general setting

$$
\Lambda_i \equiv \sum_I^{K_i} \Lambda^I; \qquad \alpha^H \equiv \frac{1}{(1-\omega_N^H)\gamma + \omega_N^H \epsilon} > 0 \qquad (24)
$$

are the good-specific relative size of the aggregate economies (those that consume good i) and a country-specific measure of weighted elasticity of consumption substitution, respectively. It is plausible that in this entangled trade network, many outputs affect country H's consumption of good h. In leading orders of importance, these include h's global supply ($\delta_{h,T}$); H's nontraded output (δ_N^H); nontraded output (δ_N^J) and traded global supply ($\delta_{j,T}$) consumed by any other country $J \in K^h$ that also consumes h; global supply ($\delta_{i,T}$) of any other traded good $i \in l^H$ consumed by H; and finally, the nontraded output (δ_N^I) and global supply ($\delta_{k,T}$) of traded goods k consumed by any country $I \in K^i$ that also consumes i.

Similar to the simpler setting of Section 2, a country's traded consumption allocation c_h^H increases with the global supply $\delta_{h,T}$, decreases with the host's nontraded output δ_N^H, and increases with nontraded output δ_N^J of all trade partners J in good h. As country H also consumes other traded variates $\{\delta_{i,T}\}_{i \in l^H}$, H's consumption c_h^H in good h tends to negatively correlate with shocks $dZ_{i,T}|_{i \neq h}$ through the substitution effect between any two traded goods. Furthermore, because the consumptions of all trade partners $J \in K_h$ in good h are tuned to the nontraded δ_N^J and traded global supplies $\{\delta_{j,T}\}_{j \in l^J}$ that they consume, these shocks are also positively compounded into c_h^H, again through trade (market clearing) and substitution effects.

Most interestingly, even in the current general trade network setting, the international transmission of output shocks follows a simple and intuitive quantitative pattern in the leading orders. That is, the transmission process involving trades in a good i with a mediating country I warrants a dampening coefficient,[29]

$$(\gamma-\epsilon)\alpha^I\frac{\Lambda^I}{\Lambda_i}=\frac{\frac{1}{\epsilon}-\frac{1}{\gamma}}{\frac{1-\omega_N^I}{\epsilon}+\frac{\omega_N^I}{\gamma}}\frac{\Lambda^I}{\Lambda_i}.$$

Here, $\frac{\Lambda^I}{\Lambda_i}$ characterizes the relative power of mediating country I in setting the global price for traded good i (through FOC), and $\left(\frac{1}{\epsilon}-\frac{1}{\gamma}\right)/\left(\frac{1-\omega_N^I}{\epsilon}+\frac{\omega_N^I}{\gamma}\right)$ quantifies how readily shocks in one consumption sector affect the others in a country.[30] Next, we examine the stochastic discount factors (SDFs) to explore how investors price the risk associated with these output shocks in different countries.

Equilibrium pricing: As shocks affect consumption allocations, they also move equilibrium prices accordingly to clear the market. The country H's log SDF is

$$\begin{aligned} m^H &= -\rho t-\gamma\sum_h^{l^H}\omega_{h,T}^H\delta_{h,T}-\gamma\omega_N^H\left[\delta_N^H-\sum_h^{l^H}(\gamma-\epsilon)\alpha^H\omega_{h,T}^H\left(1-\frac{\Lambda^H}{\Lambda_h}\right)\delta_N^H\right] \\ &- \gamma\sum_h^{l^H}\omega_{h,T}^H\sum_{J\neq H}^{K_h}(\gamma-\epsilon)\alpha^J\frac{\Lambda^J}{\Lambda_h}\left(\omega_N^J\delta_N^J+\sum_j^{l^J}\omega_{j,T}^J\delta_{j,T}\right) \end{aligned} \tag{25}$$

Reassuringly, all shocks that affect country H's consumptions are also priced by this stochastic discount factor. In particular, all traded and nontraded consumption shocks of H and any of it trade partners are compounded in m^H. As in the simpler case of Section 2, up to taste coefficients, the traded shocks are fully internationalized (in the aggregate output $\delta_{h,T}$) and spread uniformly to all countries $I\in K_h$ that consume good h. As $\omega_{h,T}^H$ generally drops with the number l^H of varieties consumed by H,[31] the country-specific traded shock of a particular variety matters even less to its country of origin in the current setting of multiple traded goods. In contrast, nontraded shocks are internalized, but not fully. As the second term within the square brackets shows, country H can tunnel its own nontraded shock in δ_N^H through trades in all l^H channels in which H participates. The ability to mitigate this shock through a particular channel h clearly decreases with a country's relative size $\frac{\Lambda^H}{\Lambda_h}$ in the world trade market for good h. Under mild home bias condition, country-

[29] As γ is (substantially) larger than ϵ, mild home bias conditions assure that $(\gamma-\epsilon)\alpha^I\frac{\Lambda^I}{\Lambda_i}<1$.

[30] Section 2 asserts that the difference $\frac{1}{\epsilon}-\frac{1}{\gamma}$ characterizes how willing a country is to substitute traded and nontraded consumptions to smooth its aggregate consumption. When this difference is large and positive as in the data, countries are flexible to make this substitution. As a result, a shock from one consumption sector is readily transmitted to the other sector. In the current setting, each country has one nontraded and several traded sectors, but all have the same pairwise substitution elasticity of ϵ.

[31] This is a consequence of the normalization condition $\omega_N^H+\sum_h^{l^H}\omega_{h,T}^H=1$.

specific nontraded shocks still matter more to the country's pricing than do the traded counterparts. Finally, we also see that traded shocks (in $\delta_{j,T}$) affecting any trade partner J are also factored in m^H. When H does not consume these goods, $j \notin l^H$, their shocks to H are similar the purely nontraded shocks of partners J.

D.2 Incomplete Market

In equilibrium, the complete financial markets equalize all countries' marginal utilities of the traded consumption and thus enable the optimal international risk sharing and consumption allocation. In reality, however, the financial markets of some countries are more developed than those of others, which should better facilitate these developed countries to manage their own as well as trade partners' output risk. Stylistically, because of either information asymmetry or lack of proper managerial enforcement, the equities associated with nontraded sectors of emerging economies are less marketable worldwide. It is interesting to explore the new qualitative implications of market incompleteness on international risk sharing and contrast them with those of the simplified complete market paradigm. To this end, we now analyze a stylized model in which nontraded output risk is the central factor behind the incompleteness in the financial markets.

Setup: We consider the world economy with perfect trades but an incomplete financial market. In the commodity sector, there are country-specific nontraded goods (one per country) and a single traded good (common to all countries). The traded good can be shipped globally without the friction, and thus only its aggregate output influences the pricing. Accordingly, we assume that the financial assets associated with the traded good sector are perfectly structured. That is, a stock S_T contingent on the aggregate output and a risk-free bond B_T paying one unit of traded good in the next period are available to investors worldwide. In contrast, the financial assets associated with nontraded sectors are incomplete. We assume that countries belong to either the "developed" or the "emerging" group. For any developed economy ($H \in \mathcal{D}$), the stock S_N^H contingent on the H's nontraded output and risk-free bond B_N^H paying one unit of H's nontraded good in the following period are also available to all investors. However, assets associated with nontraded sectors of emerging economies ($H \notin \mathcal{D}$) are not marketable and thus simply do not exist. In this framework, the world financial market is incomplete because there are more shocks than the available financial hedging instruments. To simplify the exposition, we assume a homogeneous size for all economies embedded in a two-period setting $\{t, t+1\}$, but maintain the heterogeneous consumption tastes

$\{\omega_T^H, \omega_N^H\}_H$ across countries. Relaxing all of these assumptions is tedious but straightforward.

The most convenient choice for the numeraire in this setting is the traded good, which we adopt hereafter. Thus, in every period, all prices are in (contemporaneous term of) the traded good. Because the market is incomplete, we consider the optimization problem for each country.[32] Let $x_T^{HS}, x_T^{HB}, x_N^{HFS}, x_N^{HFB}$ denote the holdings of H's investor, respectively, in world stock S_T, world bond B_T, F's stock S_N^F, and F's bond B_N^F.

$$\max_{C_{T,t}^H, x_{T,t}^{HS}, x_{T,t}^{HB}, \{x_{N,t}^{HFS}, x_{N,t}^{HFB}\}_{F\in\mathcal{D}}} U^H(C_t^H) + e^{-\rho} E_t\left[U^H(C_{t+1}^H)\right],$$

subject to the market clearing and budget constraints

$$\begin{aligned}
&\textstyle\sum_H x_{T,t}^{HS} = \sum_H x_{T,t+1}^{HS} = 1; \quad \sum_H x_{T,t}^{HS} = \sum_H x_{T,t}^{HS} = 0, \\
&\textstyle\sum_H x_{N,t}^{HFS} = \sum_H x_{N,t+1}^{HFS} = 1; \quad \sum_H x_{N,t}^{HFS} = \sum_H x_{N,t+1}^{HFS} = 0, \qquad \forall F \in \mathcal{D},
\end{aligned}$$

$$C_{T,t}^H + \Delta_{N,t}^H P_{N,t}^H \mathbb{1}_{\{H\in\mathcal{D}\}} + S_{T,t} x_{T,t}^{HS} + B_{T,t} x_{T,t}^{HB} + \sum_{F\in\mathcal{D}} S_{N,t}^F x_{N,t}^{HFS} + \sum_{F\in\mathcal{D}} B_{N,t}^F x_{N,t}^{HFB} \leq W_t^H,$$

$$C_{T,t+1}^H + \Delta_{N,t+1}^H P_{N,t+1}^H \mathbb{1}_{\{H\in\mathcal{D}\}} \leq x_{T,t}^{HS} \Delta_{T,t+1} + x_{T,t}^{HB} + \sum_{F\in\mathcal{D}} x_{N,t}^{HFS} \Delta_{N,t+1}^F + \sum_{F\in\mathcal{D}} x_{N,t}^{HFB} P_{N,t+1}^F,$$

where $C^H = \{C_T^H, C_N^H\}$ denotes the standard CES consumption aggregator as in Section 2, U^H denotes the power utility function of C^H, and W_t^H denotes investor H's initial wealth. Identity operator $\mathbb{1}_{\{F\in\mathcal{D}\}}$ equals one if F is an developed country and zero otherwise, which simply reflects the fact that investors can invest in financial assets paying nontraded goods and convert these payoffs into units of traded good at the respective nontraded price P_N^F, $\forall F \in \mathcal{D}$ for their consumption purpose. In contrast, no assets paying nontraded goods of emerging markets exist, and consequently no investors, domestic or otherwise, ever need to convert these goods into the traded good and back. In other words, in the current incomplete market setting, nontraded shocks are identical to preference shocks. Furthermore, we note that by summing all countries, the above budget constraints and market clearing conditions automatically imply the resource constraints $\sum_H C_{T,t}^H = \Delta_{T,t}$, $\sum_H C_{T,t+1}^H = \Delta_{T,t+1}$ in both periods.

First order conditions corresponding to variations about optimal holding positions x_T^{HS}, x_T^{HB},

[32] With an incomplete market, the centralized optimization can also be formulated as in Pavlova and Rigobon (2008) using the convex duality technique (Cvitanic and Karatzas (1992)). However, this approach offers an exact and analytical solution only for the special case of log utility.

x_N^{HFS}, x_N^{HFB}, respectively, generate pricing equations for all available financial assets,

$$S_{T,t} = E_t\left[\frac{M_{T,t+1}^H}{M_{T,t}^H}\Delta_{T,t+1}\right]; \qquad B_{T,t} = E_t\left[\frac{M_{T,t+1}^H}{M_{T,t}^H}\right] \qquad \forall H,$$
$$S_{N,t}^F = E_t\left[\frac{M_{T,t+1}^H}{M_{T,t}^H}\Delta_{N,t+1}^F P_{N,t+1}^F\right]; \qquad B_{N,t}^F = E_t\left[\frac{M_{T,t+1}^H}{M_{T,t}^H}P_{N,t+1}^F\right] \qquad \forall F \in \mathcal{D}, \ \forall H,$$

where $M_{T,t}^H = \frac{\partial U^H}{\partial C_{T,t}^H}$ is the *country-specific* marginal utility of the *traded* consumption[33]

In the complete market setting, the marginal utilities are necessarily equalized across countries $\frac{M_{T,t+1}^H}{M_{T,t}^H} = \frac{M_{T,t+1}^F}{M_{T,t}^F}$ $\forall\{H,F\}$, which together with market clearing conditions, then establishes directly the equilibrium consumption allocations. In the incomplete market, the marginal utilities are indirectly connected to one another only through the pricing of available assets. Accordingly, the solution approach here is very different. In sequence, we first conjecture a solution for the consumption allocations, solve for the asset prices, and verify that these prices support the conjectured consumptions in equilibrium. As before, we log-linearize the above first order conditions for all countries H and all developed countries $F \in \mathcal{D}$[34]

$$\log\left(\frac{S_{T,t}}{B_{T,t}}\right) = Cov_t\left[dm_{T,t+1}^H, \delta_{T,t+1}\right]; \qquad \log\left(\frac{S_{N,t}^F}{B_{N,t}^F}\right) = Cov_t\left[dm_{T,t+1}^H, \delta_{N,t+1}^F\right]; \tag{26}$$
$$\log B_{T,t} = E_t\left[dm_{T,t+1}^H\right] + \frac{1}{2}Var_t\left[dm_{T,t+1}^H\right]; \qquad \log\left(\frac{B_{T,t}^F}{B_{T,t}}\right) = Cov_t\left[dm_{T,t+1}^H, P_{N,t+1}^F\right],$$

where dm^H denotes the log-linearized stochastic discount factor (recall from (4) that $\alpha^H \equiv \frac{1}{\gamma\omega_T^H + \epsilon\omega_N^H}$),

$$dm_{T,t+1}^H \equiv m_{T,t+1}^H - m_{T,t}^H = \log\left(\frac{M_{T,t+1}^H}{M_{T,t}^H}\right) = -(\gamma-\epsilon)\omega_N^H d\delta_{N,t+1}^H - \frac{1}{\alpha^H}dc_{T,t+1}^H. \tag{27}$$

Equilibrium: Consistent with the log-linearization approximation scheme, we look for the equilibrium consumption allocations in the following most general log-linear form,

$$dc_{T,t+1}^H \equiv \log\left(\frac{C_{T,t+1}^H}{C_{T,t}^H}\right) = g^H + a^H d\delta_{T,t+1} + \sum_F b^{HF} d\delta_{N,t+1}^F \qquad \forall H, \tag{28}$$

[33]We recall that the current numeraire is the traded good, and therefore $M_{T,t}^H = e^{-\rho t}\omega_T^H (C_{N,t}^H)^{-\epsilon}\left[\omega_T^H (C_{T,t}^H)^{1-\epsilon} + \omega_N^H (C_{N,t}^H)^{1-\epsilon}\right]^{\frac{-\gamma+\epsilon}{1-\epsilon}}$ is the country H's pricing kernel with respect to this numeraire.

[34]Although the log-linearization technique remains useful to obtain an approximate closed-form solution, it does not address the possible multiplicity and stability of the equilibrium.

and g's, a's, b's are constant parameters to be determined, and $d\delta$'s denote the changes in log outputs, i.e., output growths ($dt = 1$)

$$d\delta_{T,t+1} \equiv \delta_{T,t+1} - \delta_{T,t} = \mu_T dt + \sigma_T dZ_T; \qquad d\delta^H_{N,t+1} \equiv \delta^H_{N,t+1} - \delta^H_{N,t} = \mu^H_N dt + \sigma^H_N dZ^H_N$$

This choice renders a log-linear SDF dm^H in the approximation and greatly simplifies the pricing of financial assets in incomplete market settings. Indeed, substituting the above conjectured consumptions and SDFs into the pricing equations and the market clearing conditions readily yields the following consumption allocations,[35] where we recall that $\alpha^I \equiv \frac{1}{\gamma\omega^I_T + \epsilon\omega^I_N} > 0$ denotes the country-specific weighted elasticity of consumption substitution.

- incomplete market: H is an emerging economy ($H \notin \mathcal{D}$)

$$c^H_{T,t} = g^H t + \frac{K\alpha^H}{\sum_I \alpha^I}\delta_{T,t} - \alpha^H \sum_{F\notin\mathcal{D}} \delta^F_{N,t} + \frac{(\gamma-\epsilon)\alpha^H}{\sum^K_I \alpha^I} \sum_{F\in\mathcal{D}} \alpha^F \omega^F_N \delta^F_{N,t}. \tag{29}$$

- incomplete market: H is a developed economy ($H \in \mathcal{D}$)

$$\begin{aligned} c^H_{T,t} \quad &= \quad g^H t + \frac{K\alpha^H}{\sum_I \alpha^I}\delta_{T,t} + \frac{\alpha^H \sum_{I\notin\mathcal{D}} \alpha^I}{\sum_{J\in\mathcal{D}} \alpha^J} \sum_{F\notin\mathcal{D}} \delta^F_{N,t} \\ &+ \quad \frac{(\gamma-\epsilon)\alpha^H}{\sum^K_I \alpha^I} \sum_{F\in\mathcal{D}} \alpha^F \omega^F_N \delta^F_{N,t} - (\gamma-\epsilon)\alpha^H \omega^H_N \delta^H_{N,t}, \end{aligned} \tag{30}$$

where g^H's are country-specific parameters. These parameters help to enforce, and thus can be found from the market clearing conditions, but because they are deterministic factors, they do not enter the analysis below. To verify these equilibrium consumptions, we substitute them back into the above pricing equations to compute all available asset prices $\{S_{T,t}, B_{T,t}\}$, $\{S^F_{N,t}, B^F_{N,t}\}_{F\in\mathcal{D}}$, which finance these consumptions by the construction of the solution. This configuration is in equilibrium,[36] because, for each available asset, the associated price is identical under all investors' perspectives in the construction. Compared with the counterpart complete market setting with a single traded good, in which the consumption allocations are[37]

[35] Specifically, the pricing equations $\log(S_{T,t}/B_{T,t})$'s determine coefficients $\{a^H\}_{\forall H}$, $\log(S^F_{N,t}/B^F_{N,t})$'s determine $\{b^{HF}\}_{\forall F\in\mathcal{D},\forall H}$, $\log B_{T,t}$'s determine $\{b^{HF}\}_{\forall F\notin\mathcal{D},\forall H}$, and $\log(B^F_{T,t}/B_{T,t})$'s determine the nontraded prices of developed countries $\{P^F_{N,t+1}\}_{\forall F\in\mathcal{D}}$.

[36] Although this is not necessarily the unique equilibrium.

[37] This is a straightforward generalization of (3) (in the basic model) to the setting where countries have heterogeneous consumption tastes (but countries' sizes are homogeneous). In the current case, the log-linearization of FOC

- complete market: $c^H_{T,t} = g^H t + \frac{K\alpha^H}{\sum_I^K \alpha^I}\delta_{T,t} + \frac{(\gamma-\epsilon)\alpha^H}{\sum_I^K \alpha^I}\sum_F \alpha^F\omega^F_N\delta^F_{N,t} - (\gamma-\epsilon)\alpha^H\omega^H_N\delta^H_{N,t}$,

the incomplete market allocations (29), (30) are markedly different in several aspects.[38] First, the traded shock impacts stay the same in both market configurations. This is because even when the market is incomplete, the equity and bond on the traded output δ_T are available to all investors, who then are able to mitigate these shocks as optimally as possible by trading these financial assets. When combined with the force of cross-country diversification in the traded sector, this result implies that country-specific traded output risks remain relatively less material to countries' risk free rates, compared with the nontraded output risk.

Second, the nontraded output shocks (in δ^F_N) of a developed country $F \in \mathcal{D}$ affect the traded consumption c^H_T of *all* other countries H similarly, regardless of the market's completeness. Because investors can trade the financial assets contingent on these nontraded shocks, their associated risk can be shared effectively. In particular, all else being equal, a surge in developed country F's nontraded output prompts F to trim its traded consumption and boosts other countries' traded consumption by forces of trades and market clearings. Similar to the complete market settings, under a mild degree of home biases, a country's own nontraded shocks matter quantitatively more to a developed country's consumption allocation than do the nontraded shocks of their *developed* trade partners.

Third, the nontraded output shocks (in δ^F_N) of an emerging country $F \notin \mathcal{D}$ are uniformly compounded in the consumptions c^H_T of all developed countries $H \in \mathcal{D}$.[39] This feature is intuitive. In the absence of financial assets in emerging markets, these shocks cannot be properly hedged. The developed investors instead opt to simply pool their consumptions uniformly to cope with the associated risk. Risk sharing is still feasible, albeit imperfect, because it is evident from the equilibrium allocation that a surge in the nontraded output from an *emerging* economy boosts traded consumptions of all *developed* economies. The coefficient characterizing this relationship, $\frac{\sum_{I\notin\mathcal{D}}\alpha^I}{\sum_{J\in\mathcal{D}}\alpha^J}$, increases (decreases) with the number of emerging (developed) economies. That is, the significance of the unhedged risk on consumption allocations is larger when the financial market is less complete in this pooling equilibrium.

Fourth, the incomplete market has a strong and surprising impact on risk sharing between

implies $m_T = -\rho t + \omega^H_T - (\gamma-\epsilon)\omega^H_N\delta^H_N + \frac{1}{\alpha^I}c^H_T$. Combining this FOC with the (log-linearized) market clearing condition (22) for traded good yields this log consumption c^H_T in complete market. See further details in appendix C.

[38] In light of the possible existence of other incomplete market settings and multiple equilibria, our discussion here pertains to the specific incomplete market setup and the associated equilibrium presented earlier in this section.

[39] That is, $\frac{\partial c^H_T}{\partial \delta^F_N}$ is same for all $F \notin \mathcal{D}$, $H \in \mathcal{D}$.

two emerging economies. Possessing no financial assets directly tied to the nontraded output shocks of their own or those of their emerging trade partners, the emerging economies also pool their traded consumption in equilibrium to uniformly share nontraded risk. Emerging country H's traded consumption c_T^H (29) decreases with not only its own nontraded good endowment δ_N^H but also with other emerging countries' nontraded output δ_N^F. The latter behavior, which is inverse to what happens in a perfect financial market, signals that the risk sharing is most severely hampered between emerging trade partners. This is indeed the group of countries whose nontraded output risk is the least hedgeable because of the incompleteness of the market.

The incomplete market setting, as formulated in this section and pertaining to the pooling equilibrium, does not qualitatively change the risk sharing behaviors, and thus prices, among developed economies. Any sizable effects stemming from market incompleteness instead arise in the group of emerging countries whose financial markets are the least developed in the setting.