Public Information and Risk-Sharing in a Pure-Exchange Economy*

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Abstract

We analyze whether the timing of public information releases affects risk-sharing and pricing in a pure exchange economy. First, information releases do not matter if agents have time additive preferences, homogeneous beliefs and access to complete markets. Second, in the case of heterogeneity in agents' beliefs, we show analytically that early information releases are Pareto improving but pricing is essentially unaffected. Third, in the case of recursive preferences we provide numerical results suggesting that early information releases improve risk-sharing, and if the EIS is large enough, they reduce the ex-ante equity premium.

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1 Introduction

There has always been a strong tendency by the regulator and the general public to demand firms to increase their transparency and release new information about corporate events as fast and frequent as possible. In addition, there are hundreds of analysts who continuously evaluate events of publicly traded firms and produce forecasts about future cash flows of these firms. Many valuations by analysts are not used to generate private information and make a profit on information asymmetries but their forecasts are made publicly available through news providers such as Bloomberg or Reuters.

The desire to generate public information builds on the perception that it leads to welfare improvements or to socially desirable redistributions of wealth within the population. We analyze under what conditions information releases are Pareto improving and how asset pricing is affected.

First, we show that expected early information releases have no value in a pure exchange economy and pricing of securities is unaffected if agents have *time additive preferences, homogeneous beliefs* and access to complete markets. The result holds for any heterogeneity in agents' subjective time discount rates and risk aversions and for any initial wealth distribution.

Second, in the case of *heterogeneous beliefs*, we prove that an early release of information unambiguously leads to a Pareto improvement in the consumption allocation. Our analytical result holds for the release of perfect or imperfect signals and for any heterogeneity in beliefs, risk aversions, time preferences and initial endowments (given agents' endowments are some constant fraction of aggregate endowment).

Intuitively, if agents disagree about the probability distribution over future endowment, they speculate. Each agent beliefs he can make a profit by betting with other agents on the realization of the state of the economy, and ex-ante every agent feels wealthier. Early resolution of uncertainty helps agents to resolve their disagreement and expect to realize their speculative profits early in time. Without early information releases agents have to wait longer until nature reveals the state of the economy and speculative profits realize. In particular, agents cannot borrow against their expected speculative profits exactly because of the disagreement. In anticipation, an early release of information allows agents to realize speculative profits and increase expected consumption relatively early in time, which in turn, increases ex-ante expected utility of every agent.[1]

Speculative trading based on subjective beliefs is common in practice. In our paper, the welfare improvement is from the standard view of agents' expectations determining their ex-ante utilities. Welfare does not necessarily improve under other criteria (Kim (2012), Brunnermeier et al. (2013), Gilboa et al. (2013)). But even when speculative trading is not found to be socially beneficial under some welfare criterion, attempting to regulate speculation can be socially suboptimal because restrictions can leave unintended adverse effects hindering markets' proper functioning as Duffie (2014) points out.[2] In this regard, we center our analysis around the traditional Pareto optimality criterion.

An early disclosure of information is particularly beneficial if the disagreement is large. Moreover, poorer and less risk averse agents benefit more from early information releases. We show that welfare implications are the same for information

[1] Our result is closely related to the literature on heterogeneous beliefs and speculation (Detemple and Murthy (1994), Zapatero (1998), Basak (2000), David (2008), Xiong and Yan (2010), Simsek (2013), Qin (2013)).

[2] Duffie (2014) argues that speculative trading is observationally inseparable from other beneficial trading motives of hedging, market making, liquidity provision and information acquisition.

releases about large and small stocks and even idiosyncratic events. The ex-ante equity premium is not essentially affected by the timing of information releases.

Hirshleifer (1971) showed that early information releases can generate market incompleteness and have negative implications for risk-sharing and welfare. We show that Hirshleifer's (1971) result is true for the case of homogeneous beliefs but does not necessarily hold if agents' beliefs are heterogeneous. In particular, we show that for certain levels of disagreement an early release of information is Pareto improving even if agents are not allowed to trade before the information is released.

Third, if agents have *recursive preferences*, numerical solutions suggest that early information releases always improve risk-sharing. Given large enough elasticity of intertemporal substitution (EIS) in the economy, an early release of information is Pareto improving. Intuitively, if information is released early the agent with the largest EIS will help the other agent to smooth his consumption over time and in return collect a premium for his service. Ex-ante both agents benefit from the trade and welfare improves. The result also holds under the stricter welfare criteria of Kim (2012), Brunnermeier et al. (2013) and Gilboa et al. (2013).

Welfare improvements are particularly large if aggregate uncertainty is large. As in the case of time additive preferences, more risk averse and poorer agents benefit more from early information releases. Moreover, information releases about large stocks have stronger welfare and pricing implications than information releases about tiny stocks.

In contrast to the case of time additive utilities, recursive utilities have a substantial impact on pricing. For large enough levels in the EIS, the ex-ante equity premium is lower if information is released early. Our results place the findings by Ross (1989) in perspective, who shows in a partial equilibrium setting that ex-ante the timing of information releases does not matter for pricing. Ross' (1989) critical assumption is an exogenous pricing kernel. We show that, in equilibrium in a heterogeneous-agent setting, consumption allocations and the pricing kernel crucially depend on the timing of information releases. Accordingly, if the pricing kernel is *endogenously* determined in equilibrium, ex-ante pricing is substantially affected by the timing of information releases. In general equilibrium, Ross' (1989) results only apply to cases of information releases on idiosyncratic events, which is in accordance with his motivating example in the introduction.

We work with a pure exchange economy because we are mostly interested in public information such as dividend and earnings announcements, announecements of corporate events like mergers and acquisitions, analyst forecasts or other firm specific news. It is reasonable to assume that these information releases only affect the information set of investors but not the set of a firm which is disclosing the information. Given a firm's information set remains unchanged as new information is released to the public (and assuming frictionless markets), the investment decisions and future cash flows of the firm are unaffected, which justifies the assumption of a pure exchange economy.[3]

Provided real investments are unaffected, it is interesting to understand whether investors in secondary markets care about these announcements and what implications information releases have on (ex-ante) risk-sharing, welfare and pricing.

Closely related to our paper is the work by Jaffe (1975), Ng (1975) and Hakansson et al. (1982) who show that in a pure exchange economy an *unexpected* release of information does not induce trade and by revealed preferences has no value if and only if agents have time additive preferences, homogeneous (prior) beliefs and markets are complete. They do not analyze ex-ante welfare and pricing implications

[3]If informaiton releases affect the investment opportunity set of a firm due to some friction, then the assumption of a pure exchange economy is unsuitable.

of *expected* information releases. In other words, they do not answer the question whether agents prefer to live in a world with early information releases or an economy with late uncertainty resolution.

In contrast, we assume that agents ex-ante *expect* that information will be disclosed early in time. In addition, an important contribution of our paper is that we are able to relax a long-standing key assumption in Jaffe (1975), Ng (1975) and Hakansson et al. (1982) about agents' initial endowments and we generalize their results in a significant way. We assume that each agent owns some constant fraction of aggregate endowment independent of the state of the economy and time, while Jaffe (1975) relies on the much stronger assumption that initial endowments equal the equilibrium consumption allocation in an economy without information disclosure.[4] Finally, in contrast to previous work, we analyze pricing of financial assets.

Hirshleifer (1971) argues that public information may improve social welfare in a *production economy* where adjustments of real investments are possible. Marshall (1974) builds on Hirshleifer's (1971) results and notes that there exist cases where information improves welfare if there is heterogeneity in beliefs across agents. In the same spirit, Jaffe and Merville (1975), Epstein and Turnbull (1980), Peng (2004), Peng and Xiong (2005), and Dang and Hakenes (2010) find that public information releases are valuable if intertemporal transfers of endowment are feasible, which effectively is a deviation from a pure exchange economy similar to assuming a production economy. Wilson (1975) suggests that information in a pure exchange economy

[4]Assuming that agents endowments are a constant fraction of aggregate endowment independent of the state of the world and time is important for two reasons. First, it is one of the sufficient conditions we use to ensure the uniqueness of the equilibrium. Second, it ensures our analytical results (early information releases are Pareto improving) are unambiguous. If agents' endowments are not constant fractions of aggregate endowment, then there exists no unambiguous answer.

does not improve welfare but he notes that information can introduce economies of scales (see also Gonedes (1975) and Stigler (1962) for a discussion on the value of information for operating decisions in a firm).

In the context of *information asymmetries*, Hirshleifer (1971) and Fama and Laffer (1971) argue that in a pure exchange economy private information may lead to large redistributions of wealth from uninformed to informed agents. If information production is costly, then private information acquisitions result in socially sub-optimal outcomes. The release of public information (if it is not too costly) may preclude acquisitions of private information and a socially sub-optimal allocation may be prevented (and a second-best allocation is achieved). Berk (1993) shows that with enough heterogeneity there is at least one agent who wants to generate public information in order to make markets incomplete. Berk (1993) does not discuss welfare implications in his model.

The paper is organized as follows. Section 2 introduces the basic structure of our model. Section 3 studies welfare and pricing implications of early information releases in the case of time additive preferences and both homogeneity and heterogeneity in beliefs. We provide rigorous proofs for most of our results in section 3. Section 4 numerically solves a model with recursive preferences and illustrate how early information releases can be Pareto improving and substantially affect asset pricing. Finally, section 5 concludes. The Appendix contains all the remaining proofs.

2 Model

We consider a pure exchange economy with three dates, $t \in \{-1, 0, 1\}$. The economy is endowed with a single consumption good which we choose to be the numeraire. There are two types of agents, $i \in I = \{A, B\}$. Agents A and B differ with respect to their preference specifications, beliefs and initial endowments. There is no information asymmetry between agents and differences in beliefs are understood as an agreement to disagree about future endowment. We assume perfect competitive markets and all agents are price-takers.

2.1 Information

Let $(\mathbb{S}, \mathcal{F}_1)$ be a measureable space with the finite set $\mathbb{S}$ consisting of $N > 1$ elementary events and the σ-algebra $\mathcal{F}_1 = 2^{\mathbb{S}} = \{k : k \subseteq \mathbb{S}\}$ is the power set of $\mathbb{S}$. We define the filtration $\mathcal{F} = \{\mathcal{F}_t\}_{t\in\{-1,0,1\}}$, a non-decreasing sequence of σ-algebras $\mathcal{F}_{-1} \subseteq \mathcal{F}_0 \subseteq \mathcal{F}_1$ with $\mathcal{F}_{-1} = \{\emptyset, \mathbb{S}\}$. Filtration $\mathcal{F}$ is common to all agents and describes the the amount and quality of released public information or in other words, the timing of uncertainty resolution in the economy.

In the following we compare equilibrium allocations across economies which differ with respect to the fineness of the σ-algebra at time 0, that is, the amount of public information available at time 0. We are interested in comparative statics between a benchmark economy with the filtration $\mathcal{F}$ such that $\mathcal{F}_0 = \mathcal{F}_{-1}$ ($\mathcal{F}_{-1}$ and $\mathcal{F}_1$ as defined above) and a second economy with the filtration $\widehat{\mathcal{F}}$ such that $\widehat{\mathcal{F}}_{-1} = \mathcal{F}_{-1}$ and $\widehat{\mathcal{F}}_0 = \widehat{\mathcal{F}}_1 = \mathcal{F}_1$. The benchmark economy with filtration $\mathcal{F}$ is understood as a world where no information about state $s \in \mathbb{S}$ is acquired between time -1 and 0. We call the benchmark case the economy without early information relesases or the economy with late uncertainty resolution. The second case with filtration $\widehat{\mathcal{F}}$ is an economy where a perfect signal revealing state $s \in \mathbb{S}$ is observed at time 0. We call the case with filtration $\widehat{\mathcal{F}}$ the economy with early information releases or the economy with early uncertainty resolution.

We define $\mathcal{L}$ as the set of processes adapted to the filtration $\{\mathcal{F}_t\}_{t\in\{0,1\}}$, and $\mathcal{L}_+$

as the set of strictly positive processes of $\mathcal{L}$.[5] The sets $\widehat{\mathcal{L}}$ and $\widehat{\mathcal{L}}_+$ corresponding to the filtration $\left\{\widehat{\mathcal{F}}_t\right\}_{t\in\{0,1\}}$ are defined analogously. Notice that $\mathcal{L} \subset \widehat{\mathcal{L}}$ (and $\mathcal{L}_+ \subset \widehat{\mathcal{L}}_+$) because $\mathcal{F}_0 \subset \widehat{\mathcal{F}}_0$.

In the following, we write every variable corresponding to the economy with filtration $\mathcal{F}$ and $\widehat{\mathcal{F}}$ with either no indication or a "hat", respectively.

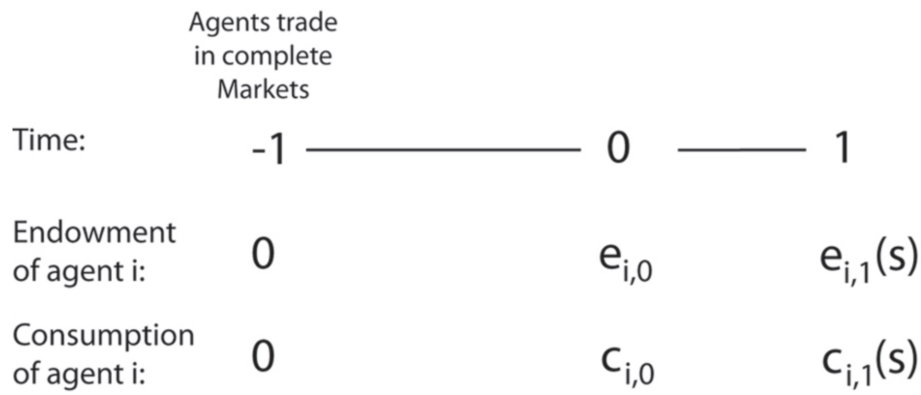
Figure 1 illustrates the main idea of our paper. The two diagrams describe economies with late versus early uncertainty resolution. Endowments $e_{i,0}, e_{i,1}(s)$ are identical in the two economies.[6] However, in the economy with early information releases (bottom diagram) agent $i \in I$ appears to have more flexibility with respect to his consumption choice at time 0 than in the economy with late uncertainty resolution (top diagram). In the bottom diagram agent $i \in I$ can choose his consumption $\widehat{c}_{i,0}(s)$ at time 0 to be state-dependent because state $s \in \mathbb{S}$ is revealed just before time 0. In the top diagram $c_{i,0}$ has to be state-independent because state $s \in \mathbb{S}$ has not yet been observed at time 0.

Market clearing puts some discipline on the flexibility of $\widehat{c}_{i,0}(s)$ because equilibrium requires aggregate consumption and supply to be equal. Clearly, in the case of identical agents we will have $\widehat{c}_{A,0}(s) = \widehat{c}_{B,0}(s)$ and market clearing forces $\widehat{c}_{i,0}(s) = c_{i,0}$ to be state-independent since aggregate endowment at time 0 is also constant across states. Early information releases will not matter in the case of identical agents. In contrast, if agents A and B differ with respect to their preferences or beliefs, then it is not implausible that $\widehat{c}_{A,0}(s) \neq \widehat{c}_{B,0}(s)$ with $\sum_{i\in\{A,B\}}(\widehat{c}_{i,0}(s) - e_{i,0}) = 0$. If $\widehat{c}_{i,0}(s)$ is state-dependent, then public information appears to improve risk-sharing in

[5] Let $\mathcal{B}$ be the σ-algebra of Borel subsets of $\mathbb{R}$. Function $x_t(s) \in \mathbb{X}_t \subseteq \mathbb{R} : \mathbb{S} \to \mathbb{R}$ is called $\mathcal{F}_t/\mathcal{B}$ measurable if $\{s : x_t(s) \in X_t\} \in \mathcal{F}_t$ for any $X_t \in \mathcal{B}$. A process $x = \{x_t\}_{t\in\{0,1\}}$ is adapted to the filtration $\{\mathcal{F}_t\}_{t\in\{0,1\}}$ if x_t is a $\mathcal{F}_t/\mathcal{B}$ measurable function $x_t : \mathbb{S} \to \mathbb{R}$, $\forall t \in \{0,1\}$.

[6] See below for details about the endowment and consumption processes.

No Public Signal/ Late Uncertainty Resolution:



Trade before a Perfect Signal/ Early Uncertainty Resolution:

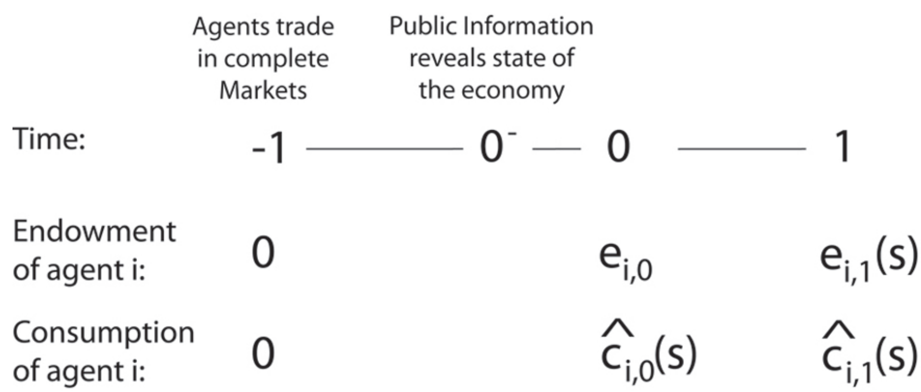


Figure 1: Comparison between two economies: (i) economy with no public information releases or late uncertainty reslution (top diagram), and (ii) economy with an early release of a perfect public signal or early uncertianty resolution (bottom diagram). The two economies only differ with respect to consumption at time 0. In the bottom diagram agents know the state of the world $s \in \mathbb{S}$ at time 0 and are able to choose consumption to be state-dependent. In the top diagram agents do not get any information about the state of the world until time 1 and have to choose consumption at time 0 to be state-independent.

the economy and there are interesting implications for the equilibrium consumption allocation and pricing of assets.

2.2 Endowments, Financial Markets and Budget Constraints

The endowment of agent $i \in I$ is denoted by the adapted process $e_i = \{e_{i,t}\}_{t\in\{0,1\}} \in \mathcal{L}_+$. Agents are not endowed with any consumption goods at time -1. There is no uncertainty about the endowment at time 0 and, independent of $s \in \mathbb{S}$, agent $i \in I$ is endowed with $e_{i,0} \in \mathbb{R}_+$ units of the consumption good. At time 1, the endowment of agent $i \in I$ in state $s \in \mathbb{S}$ is $e_{i,1}(s) \in \mathbb{R}_+$. Aggregate endowment is $e = \{e_t\}_{t\in\{0,1\}} = \sum_{i\in\{A,B\}} e_i$, that is, at time 0 and 1 in state $s \in \mathbb{S}$ the economy is endowed with $e_0 = \sum_{i\in\{A,B\}} e_{i,0}$ and $e_1(s) = \sum_{i\in\{A,B\}} e_{i,1}(s)$ units of the consumption good. We assume throughout the paper that agents' endowments are proportional across time and states, $\frac{e_{A,0}}{e_{B,0}} = \frac{e_{A,1}(s)}{e_{B,1}(s)}$ $\forall s \in \mathbb{S}$. Proportional endowments ensure that agents' relative initial wealths are independent of prices. This assumption is one of the sufficient conditions we impose to ensure uniqueness of our equilibrium.

Working with an endowment economy implicitly assumes that a firm's investment policy is independent of the filtration in the economy. This is a reasonable assumption if we are thinking of public information such as earnings and dividend announcements, which are unlikely to change the information set of a firm. The assumption is less susitable if one considers for instance news about macroeconomic forcasts, which are likely to change the information set of a firm and lead to an adjustment of a firm's investments and cash flows (see for instance Hirshleifer (1971) for a discussion on the value of information in a production economy).

Agents have access to trade a complete set of Arrow-Debreu securities at time

-1.[7] Let $\mathcal{P}_t$ be the partition consisting of all the atoms of $\mathcal{F}_t$, for $t \in \{0,1\}$. Notice that $\mathcal{P}_1 = \mathbb{S}$. Arrow-Debreu security $AD_{t,k}$ pays 1 at time $t \in \{0,1\}$ if event $k \in \mathcal{P}_t$ realizes, and 0 otherwise. The price of $AD_{t,k}$ at time -1 is denoted by $q_{t,k} \in \mathbb{R}_+$, which is determined in equilibrium. Let q be the collection of all Arrow-Debreu security prices $q_{t,k}$ $\forall k \in \mathcal{P}_t, t \in \{0,1\}$. Because we assume no consumption at time -1, we need a normalization for the Arrow-Debreu security prices. We impose the usual condition $\sum_{k \in \mathcal{P}_0} q_{0,k} = 1$, or in other words, we set the risk-free interest rate between date -1 and 0 equal to zero. $\sum_{k \in \mathcal{P}_0} q_{0,k} = 1$ means that we choose a claim to 1 unit of the consumption good at time 0 and independent of state $s \in \mathbb{S}$ as the numeraire.

Given complete markets and the Arrow-Debreu security prices q, we can price any security in the economy. We use the common definition that the stock market is a claim to aggregate endowment. Since endowment is state-independent at time 0, all interesting stock market pricing implications derive from the risky dividend paid at time 1. We focus on the pricing of a dividend strip with a claim to aggregate endowment at time 1. At time -1, the price of the dividend strip is

$$P_{-1,1} = \sum_{s \in \mathbb{S}} q_{1,s} e_1(s). \tag{1}$$

The price at time -1 of a zero-coupon bond with face value 1 and maturity at date 1 is

$$B_{-1,1} = \frac{1}{1 + r_{-1,1}} = \sum_{s \in \mathbb{S}} q_{1,s}. \tag{2}$$

The equity premium of the dividend strip between time -1 and 1 under agnet $i \in I$'s

[7] Complete markets means that there exists a full set of securities $AD_{k,t}$ $\forall k \in \mathcal{P}_t, t \in \{0,1\}$.

belief is

$$\begin{aligned} ep^i_{-1,1} &= \ln\left(\frac{E^i\left[e_1\left(s\right)\right]}{P_{-1,1}}\frac{1}{1+r_{-1,1}}\right) \\ &= \ln\left(E^i\left[e_1\left(s\right)\right]\right) - \ln\left(\frac{E^i\left[\frac{q_{1,s}}{\mathbb{P}_i(s)}e_1\left(s\right)\right]}{E^i\left[\frac{q_{1,s}}{\mathbb{P}_i(s)}\right]}\right), \end{aligned} \tag{3}$$

where the operator $E^i\left[x\left(s\right)\right] = \sum_{s\in\mathbb{S}}\mathbb{P}_i\left(s\right)x\left(s\right)$ is the expectation of function $x\left(s\right):\mathbb{S}\to\mathbb{R}$ over the state space $\mathbb{S}$ under agent $i\in I$'s beliefs. Function $\mathbb{P}_i:\mathbb{S}\to(0,1]$ with $\sum_{s\in\mathbb{S}}\mathbb{P}_i\left(s\right)=1$ denotes the probability measure specific to agent $i\in I$ and captures his beliefs. By construction of $\mathbb{P}_i$ ($\mathbb{P}_i\left(s\right)>0\ \forall s\in\mathbb{S}$), measures $\mathbb{P}_A$ and $\mathbb{P}_B$ are absolutely continuous with respect to each other.

We denote the consumption plan of agent $i\in I$ by $c_i=\left\{c_{i,t}\right\}_{t\in\{0,1\}}\in\mathcal{L}_+$. Since aggregate endowment at time -1 is zero, there is also no consumption at time -1. Agent $i\in I$'s set of admissible consumption plans is defined as

$$\mathbb{B}_i=\left\{c_i\in\mathcal{L}_+ : 0\geq\sum_{k\in\mathcal{P}_0}q_{0,k}\left(c_{i,0}\left(k\right)-e_{i,0}\right)+\sum_{s\in\mathbb{S}}q_{1,s}\left(c_{i,1}\left(s\right)-e_{i,1}\left(s\right)\right)\right\}. \quad (BC)$$

For the alternative economy with filtrations $\widehat{\mathcal{F}}$ we assume that $\widehat{\mathbb{P}}_i$ is identical to $\mathbb{P}_i$ and we also set $\widehat{e}_i=e_i$, $\forall i\in\{A,B\}$. The variables $\widehat{\mathcal{P}}_t$, $\widehat{AD}_{t,k}$, $\widehat{q}_{t,k}$, $\widehat{q}$, $\widehat{P}_{-1,1}$, $\widehat{B}_{-1,1}$, $\widehat{ep}^i_{-1,1}$, $\widehat{c}_i$, $\widehat{\mathbb{B}}_i$ $\forall k\in\widehat{\mathcal{P}}_t$, $t\in\{0,1\}$ corresponding to $\widehat{\mathcal{F}}$ are defined analogously to $\mathcal{P}_t$, $AD_{t,k}$, $q_{t,k}$, q, $P_{-1,1}$, $B_{-1,1}$, $ep^i_{-1,1}$, c_i, $\mathbb{B}_i$ $\forall k\in\mathcal{P}_t$, $t\in\{0,1\}$.

2.3 Preferences, Beliefs and Optimal Consumption Choice

Agent $i\in I$'s objective function is specified by the homothetic function $U_i\left(c_i;\mathbb{P}_i\right):\mathbb{B}_i\to\mathbb{R}_+$, with $\frac{\partial U_i(c_i;\mathbb{P}_i)}{\partial c_{i,t}(k)}>0$ and $\frac{\partial^2 U_i(c_i;\mathbb{P}_i)}{\partial\left(c_{i,t}(k)\right)^2}<0\ \forall k\in\mathcal{P}_t, t\in\{0,1\}$. The assumption of homothetic preferences is relatively weak. It is an important assumption for our analysis because it is one of the sufficient conditions we impose to ensure uniqueness of the equilibrium.

Given the prices q, agent $i \in I$ chooses $c_i \in \mathbb{B}_i$ to maximize utility,

$$\sup_{\{c_i \in \mathbb{B}_i\}} \{U_i(c_i; \mathbb{P}_i)\}. \tag{$P1$}$$

Problem $(P1)$ has a unique solution which we denote by $c_i^\diamond(q)$ and $\lambda_i^\diamond(q)$, where $\lambda_i^\diamond(q)$ is the Lagrange multiplier given the prices q. The optimal consumption plan $c_i^\diamond(q)$ and the multiplier $\lambda_i^\diamond(q)$ satisfy the Karush-Kuhn-Tucker (KKT) optimality conditions (23) through (27) in the Appendix.

The assumption of locally non-satiated preferences implies that the budget constraint (26) evaluated at $c_i^\diamond(q)$ holds with equality, and $\lambda_i^\diamond(q) > 0$. Summing equation (23), which determines state prices q, over $k \in \mathcal{P}_0$ and using our normalization of prices $\sum_{k \in \mathcal{P}_0} q_{0,k} = 1$ implies

$$\lambda_i^\diamond(q) = \sum_{k \in \mathcal{P}_0} \frac{\partial U_i(c_i^\diamond(q); \mathbb{P}_i)}{\partial c_{i,0}(k)}, \tag{4}$$

which is positive.

The variables $\widehat{c}_i^\diamond(\widehat{q})$, $\widehat{\lambda}_i$, $\widehat{\lambda}_i^\diamond(q)$ corresponding to $\widehat{\mathcal{F}}$ are defined analogously to $c_i^\diamond(q)$, λ_i, $\lambda_i^\diamond(q)$.

2.4 Equilibrium

Our goal is to solve for the general equilibrium Arrow-Debreu security prices q^*. The Walrasian equilibrium concept is suitable in our context because we work with complete markets and the time and state space is finite. A Walrasian equilibrium is a collection (c_A^*, c_B^*, q^*) such that $\sum_{i \in \{A,B\}} c_i^* = \sum_{i \in \{A,B\}} e_i$ (consumption market clearing) with $c_i^* = c_i^\diamond(q^*)$ (optimal consumption is a solution to problem $(P1)$ or equivalently satisfies conditions (23) through (27) given the prices q^*). Similarly, we denote the equilibria corresponding to $\widehat{\mathcal{F}}$ by the collection $(\widehat{c}_A^*, \widehat{c}_B^*, \widehat{q}^*)$.

In our model there exists a globally unique equilibrium (see Appendix). Global uniqueness is important in our analysis because in the case of multiple equilibria it is impossible to compare equilibrium allocations $\{c_i^*\}_{i\in\{A,B\}}$ and $\{\widehat{c}_i^*\}_{i\in\{A,B\}}$ and prices q^* and $\widehat{q}^*$ in any sensible way.

We define the utility of a representative agent as

$$U\left(c_A, c_B; \mathbb{P}_A, \mathbb{P}_B\right) = U_A\left(c_A; \mathbb{P}_A\right) + \lambda U_B\left(c_B; \mathbb{P}_B\right), \tag{5}$$

with the Pareto weight $\lambda = \frac{\lambda_A}{\lambda_B}$. Following the first and second Theorem of Welfare Economics the equilibrium allocation $\left(\{c_i^*\}_{i\in\{A,B\}}, q^*\right)$ is equivalent to the solution of a benevolent social planer's optimization of $U\left(c_A, c_B; \mathbb{P}_A, \mathbb{P}_B\right)$ with respect to c_A, c_B and λ, subject to the market clearing condition $\sum_{i\in\{A,B\}} c_i^* = \sum_{i\in\{A,B\}} e_i$ and agent A's budget constraint (26) with the Arrow-Debreu security prices q^* given by equations (23), (24) and (4).

3 Time Additive Preferences

In the case of time additive preferences we write the utility of agent $i \in I$ as

$$U_i\left(c_i; \mathbb{P}_i\right) = E^i\left[\sum_{t\in\{0,1\}} \beta_i^t u_i\left(c_{i,t}\left(s\right)\right)\right], \tag{6}$$

where $\beta_i^t u_i\left(c_{i,t}\left(s\right)\right)$ is the utility he receives at time $t \in \{0,1\}$ in state $s \in \mathbb{S}$. The utililty of the representative agent (5) can be written as

$$U\left(c_A, c_B; \mathbb{P}_A, \mathbb{P}_B\right) = \sum_{t\in\{0,1\}} E^A\left[\beta_A^t u_A\left(c_{A,t}\left(s\right)\right) + \lambda \frac{\mathbb{P}_B\left(s\right)}{\mathbb{P}_A\left(s\right)} \beta_B^t u_B\left(c_{B,t}\left(s\right)\right)\right].$$

c_A^*, c_B^*, q^* and λ^* in the economy with no early information releases (filtration $\mathcal{F}$) satisfy the system of equations (28) through (33) in the Appendix. $\widehat{c}_A^*$, $\widehat{c}_B^*$, $\widehat{q}^*$ and $\widehat{\lambda}^*$ in the economy with early information releases (filtration $\widehat{\mathcal{F}}$) satisfy the system of equaitons (34) through (38) in the Appendix. The two systems are identical except for equations (28) and (34) at $t = 0$.

3.1 Homogeneity in Beliefs

If agents hold identical beliefs about the probability distribution over future endowments, $\mathbb{P}_A \equiv \mathbb{P}_B$, the first order condition (FOC) (34) at $t = 0$ in the economy where a perfect signal is observed at time 0 becomes

$$\frac{\partial U\left(\widehat{c}_{A,0}^*(s)\right)}{\partial \widehat{c}_{A,0}^*(s)} = \widehat{\lambda}^* \frac{\partial U\left(\widehat{c}_{B,0}^*(s)\right)}{\partial \widehat{c}_{B,0}^*(s)}, \quad \forall s \in \mathbb{S}. \tag{7}$$

Because agents are risk averse, marginal utilities are monotonically decreasing in consumption. If agent A's equilibrium consumption in state $s_1 \in \mathbb{S}$ is higher than in state $s_2 \in \mathbb{S}\backslash\{s_1\}$ equation (7) implies that agent B's consumption in state s_1 is also higher than in state s_2,

$$\widehat{c}_{A,0}^*(s_1) > \widehat{c}_{A,0}^*(s_2) \Longleftrightarrow \widehat{c}_{B,0}^*(s_1) > \widehat{c}_{B,0}^*(s_2), \quad \forall (s_1, s_2) \in \mathbb{S} \times \mathbb{S},\ s_1 \neq s_2.$$

However, this clearly violates the market clearing condition (35) at time 0. As a result, consumption of both agents at time 0, $\widehat{c}_{A,0}^*(s)$ and $\widehat{c}_{B,0}^*(s)$ must be state-independent.[8] Equations (28) through (33), and (34) through (38) are structurally

[8]Alternatively, because markets are complete, Pareto weight $\widehat{\lambda}^*$ does not depend on the state of the world. When agents have homogeneous beliefs, the system of equations (34) and market clearing (35) at time 0 in the economy with early uncertainty resolution has no explicit state dependence,

$$\frac{\partial U(\widehat{c}_{A,0}^*(s))}{\partial \widehat{c}_{A,0}^*(s)} = \widehat{\lambda}^* \frac{\partial U(\widehat{c}_{B,0}^*(s))}{\partial \widehat{c}_{B,0}^*(s)}; \qquad \widehat{c}_{A,0}^*(s) + \widehat{c}_{B,0}^*(s) = e_0.$$

Consequently, its solutions are state-independent.

identical in the two economies with and without early information releases, if $\widehat{c}_{i,0}^*(s)$ is state-independent for all $i \in I$. Accordingly, the equilibrium consumption allocations in the two economies are identical, $c_i^* = \widehat{c}_i^*$, $\forall i \in I$.[9] We recapitulate this information-irrelevance result in the following proposition.

Proposition 1 *(Irrelevance of information) When agents have homogeneous beliefs about the probability distribution over future endowments and markets are complete, the equilibrium consumption allocation and expected utilities of all agents are identical in two economies which only differ with respect to the timing of uncertainty resolution. In other words, early information releases about future uncertain endowments do not matter and have no value in an environment with homogeneity in beliefs.*

Intuitively, when agents agree on the prospect of the economy they can optimally share risk by trading a complete set of state contingent contracts and there is no doubt or disagreement about the "fundamental value" of these contracts. Early information releases may be relevant and possibly welfare-improving only if agents disagree on the porbability distribution over future endowments and the "fundamental value" of Arrow-Debreu securities, asset market are incomplete, or agents have non-time-separable preferences.

This result and intuition are related to the results of Jaffe (1975), Ng (1975) and Hakansson et al. (1982), who show that agents do not wish to trade after an unexpected release of information if (i) agents are initially endowed with precisely the equilibrium consumption allocation at the onset, $e_i = c_i^*$, $\forall i \in I$, and (ii) agents have homogeneous beliefs. Their no-trade result is restrictive because of the stark

[9] Alternatively, it is easy to see that the allocation c_A^*, c_B^* satisfies equations (34) through (38) if $\mathbb{P}_A \equiv \mathbb{P}_B$. Since the equilibrium is globally unique, the equilibria in the two economies with and without early information releases coincide.

assumption about initial endowments. In contrast, Proposition 1 asserts that even when agents start out with *arbitrary* endowments, an expected release of a perfect signal at time 0 about the future state of the economy does not affect the equilibrium consumption allocation and ex-ante expected utility of any agent.

In summary, public information disclosures do not matter in our setting. First, we show that an expected early release of information does not improve risk-sharing and ex-ante expected utility. Second, we know from Jaffe (1975) that an unexpected release of information does not induce trading either.

3.2 Heterogeneity in Beliefs and Speculation

3.2.1 Special Case of Log-Utility

In the special case of log-utility we are able to derive explicit closed-form solutions for the equilibrium consumption allocation and agents' utilities. The explicit solutions make it is easy to understand how and why the release of public information improves risk-sharing and expected utilities of all agents.

In Proposition 2 we derive explicit solutions in the economy with early and late uncertainty resolution (filtration $\widehat{\mathcal{F}}$ respectively $\mathcal{F}$).

Proposition 2 *Suppose $u_i(x) = \ln(x), \forall i \in I$. In an economy without information releases between time -1 and 0 (filtration $\mathcal{F}$) agent $i \in I$'s ex-ante expected utility is*

$$U_i(c_i^*; \mathbb{P}_i) = \ln(e_0) + \beta_i E^i[\ln(e_1(s))] - \ln\left(1 + \frac{e_{j,0}(1+\beta_i)}{e_{i,0}(1+\beta_j)}\right) - \beta_i E^i\left[\ln\left(1 + \frac{e_{j,0}\beta_j(1+\beta_i)\mathbb{P}_j(s)}{e_{i,0}\beta_i(1+\beta_j)\mathbb{P}_i(s)}\right)\right], \quad \forall i,j \in I, i \neq j. \tag{8}$$

In an economy where a perfect signal about the future state of the economy is observed *at time 0 (filtration $\widehat{\mathcal{F}}$) agent $i \in I$'s ex-ante expected utility is*

$$U_i\left(\widehat{c}_i^*; \mathbb{P}_i\right) = \ln\left(e_0\right) + E^i\left[\beta_i \ln\left(e_t\left(s\right)\right)\right] - E^i\left[\ln\left(1 + \frac{e_{j,0}\left(1+\beta_i\right)\mathbb{P}_j\left(s\right)}{e_{i,0}\left(1+\beta_j\right)\mathbb{P}_i\left(s\right)}\right)\right]$$
$$- \beta_i E^i\left[\ln\left(1 + \frac{e_{j,0}\beta_j\left(1+\beta_i\right)\mathbb{P}_j\left(s\right)}{e_{i,0}\beta_i\left(1+\beta_j\right)\mathbb{P}_i\left(s\right)}\right)\right], \quad \forall i, j \in I, i \neq j. \tag{9}$$

Welfare is unambiguously larger in the economy with early information releases,

$$U_i\left(\widehat{c}_i^*; \mathbb{P}_i\right) \geq U_i\left(c_i^*; \mathbb{P}_i\right), \forall i \in I.$$

Expected utilities in the two economies $U_i\left(c_i^*; \mathbb{P}_i\right)$ and $U_i\left(\widehat{c}_i^*; \mathbb{P}_i\right)$ only differ with respect to the third terms in equations (8) and (9). Jensen's inequality tells us that

$$E^i\left[\ln\left(1 + \frac{e_{j,0}\left(1+\beta_i\right)\mathbb{P}_j\left(s\right)}{e_{i,0}\left(1+\beta_j\right)\mathbb{P}_i\left(s\right)}\right)\right] \leq \ln\left(E^i\left[1 + \frac{e_{j,0}\left(1+\beta_i\right)\mathbb{P}_j\left(s\right)}{e_{i,0}\left(1+\beta_j\right)\mathbb{P}_i\left(s\right)}\right]\right)$$
$$= \ln\left(1 + \frac{e_{j,0}\left(1+\beta_i\right)}{e_{i,0}\left(1+\beta_j\right)}\right),$$

which proves the result $U_i\left(\widehat{c}_i^*; \mathbb{P}_i\right) \geq U_i\left(c_i^*; \mathbb{P}_i\right), \forall i \in I$ in Proposition 2.

In equation (8) only the fourth term depends on the disagreement, $\frac{\mathbb{P}_A}{\mathbb{P}_B}$. Jensen's inequality implies

$$\beta_i E^i\left[\ln\left(1 + \frac{e_{j,0}\beta_j\left(1+\beta_i\right)\mathbb{P}_j\left(s\right)}{e_{i,0}\beta_i\left(1+\beta_j\right)\mathbb{P}_i\left(s\right)}\right)\right] \leq \beta_i \ln\left(1 + \frac{e_{j,0}\beta_j\left(1+\beta_i\right)}{e_{i,0}\beta_i\left(1+\beta_j\right)}\right),$$

where the latter term is understood as a case if there was no disagreement at all. Intuitively, every agent's utility increases due to speculation at time 1.[10] In contrast, in equation (9) also the third term depends on disagreement $\frac{\mathbb{P}_A}{\mathbb{P}_B}$. Since the true state of the world will be observed already at time 0, agents can speculate already at time 0. Our derivation shows that an early release of public information at time 0

[10] Notice that we can associate the third and fourth terms in equations (8) and (9) with utility derived from consumption at time 0 respectively time 1.

increases utilities of all agents because agents can speculate on their beliefs already at time 0 instead of only at time 1.

Disagreement about the probability distribution over future endowmens implies disagreement about the "fundamental value" of contingent claims. Agents speculate on their beliefs and ex-ante all agents feel wealthier and are better off in terms of expected utility. Each agent believes that his trading counterparty has wrong beliefs and given the respective probability distribution function each agent expects to make a profit off the other agents. If the state of the world is revealed at time 1 (no early release of public information), expected profits from speculation are paying off only at time 1 and agents cannot borrow against the expected wealth from speculation because of the disagreement. In this case of late uncertainty resolution agents can only increase their expected consumption at time 1 but not at time 0. In contrast, if a public signal is known to reveal the state of the world already at time 0, then agents expect to realize their profits from speculation at time 0 and are able to increase both their expected consumption at time 0 and time 1. An early release of information improves expected utilities of all agents (Pareto improvement).

Jensen's inequality further suggests that an early release of public information is relatively more valuable if the disagreement is relatively large. Large disagreement means that the ratio $\frac{\mathbb{P}_A(s)}{\mathbb{P}_B(s)}$ varies a lot from 1 and across the state space $\mathbb{S}$. Intuitively, it makes sense that agents expect larger speculative profits if they believe their speculative counterparty is very wrong rather than if they were in a situation where agents only slightly disagree.

Early information releases are Pareto improving if expectations are evaluated under agents' respective beliefs. In contrast, if welfare is assessed according to the criterion introduced by Kim (2012) and Brunnereier et al. (2013), the economy with no early information releases is preferred as it reduces speculative trading. Although these papers make an interesting point, these criteria are equivalent to basically saying that agents maximize "wrong" objective functions and a social planner should ignore agents' preferences and beliefs.

3.2.2 General case of Power Utility

We are able to generalize the main results of the previous section to a general case of power utility and heterogeneity in relative risk aversion (RRA). We first establish a simple relation between wealth and expected utility for the case of constant relative risk aversion (CRRA) and proportional endowments. We begin with the economy with early information releases. Let $\widehat{U}_{i,t}$ denote the expected utility agent $i \in I$ expects to receive at time $t \in \{0,1\}$ in equilibrium in the economy with early information releases,

$$\begin{aligned} \widehat{U}_{i,t} &\equiv E^i\left[\beta_i^t u_i\left(\widehat{c}^*_{i,t}(s)\right)\right], \qquad \forall t \in \{0,1\} \qquad (10) \\ U_i\left(\widehat{c}^*_i;\mathbb{P}_i\right) &= \sum_{t\in\{0,1\}} \widehat{U}_{i,t}. \end{aligned}$$

Agent $i \in I$'s initial wealth which finances his optimal consumption plan is

$$\begin{aligned} \widehat{W}_i &= \sum_{s\in\mathbb{S}}\left(\widehat{q}^*_{0,s}\widehat{c}^*_{i,0}(s) + \widehat{q}^*_{1,s}\widehat{c}^*_{i,1}(s)\right) \\ &= \frac{1-\gamma_i}{\sum_{s\in\mathbb{S}}\mathbb{P}_i(s)\frac{\partial u\left(\widehat{c}^*_{i,0}(s)\right)}{\partial \widehat{c}_{i,0}(s)}} U_i\left(\widehat{c}^*_i;\mathbb{P}_i\right). \qquad (11) \end{aligned}$$

In the second equality we have used equation (37) and the explicit specification of CRRA preferences. Combining the initial wealth expressions (11) for agents A and

B yields the initial wealth ratio

$$\begin{aligned}\frac{\widehat{W}_A}{\widehat{W}_B} =& \frac{(1-\gamma_A)}{(1-\gamma_B)} \frac{\sum_{s\in\mathbb{S}} \mathbb{P}_B(s) \frac{\partial u(\widehat{c}^*_{B,0}(s))}{\partial \widehat{c}_{B,0}(s)}}{\sum_{s\in\mathbb{S}} \mathbb{P}_A(s) \frac{\partial u(\widehat{c}^*_{A,0}(s))}{\partial \widehat{c}_{A,0}(s)}} \frac{U_A(\widehat{c}^*_A;\mathbb{P}_A)}{U_B(\widehat{c}^*_B;\mathbb{P}_B)} \\ =& \frac{(1-\gamma_A)}{(1-\gamma_B)} \frac{1}{\widehat{\lambda}^*} \frac{U_A(\widehat{c}^*_A;\mathbb{P}_A)}{U_B(\widehat{c}^*_B;\mathbb{P}_B)}, \end{aligned} \tag{12}$$

where in the last equality we made use of the FOC (34) at $t=0$.

Similarly, we define $U_{i,t} \equiv E^i\left[\beta_i^t u_i\left(c^*_{i,t}(s)\right)\right], \forall t \in \{0,1\}$ as the expected utility agent $i \in I$ expects to receive at time $t \in \{0,1\}$ in equilibrium in the economy with late uncertainty resolution. By identical derivation we have,

$$\frac{W_A}{W_B} = \frac{(1-\gamma_A)}{(1-\gamma_B)} \frac{1}{\lambda^*} \frac{U_A(c^*_A;\mathbb{P}_A)}{U_B(c^*_B;\mathbb{P}_B)}. \tag{13}$$

Together, the two wealth ratios (12) and (13) imply

$$\frac{\frac{U_A\left(c^*_A;\mathbb{P}_A\right)}{U_B\left(c^*_B;\mathbb{P}_B\right)}}{\frac{U_A\left(\widehat{c}^*_A;\mathbb{P}_A\right)}{U_B\left(\widehat{c}^*_B;\mathbb{P}_B\right)}} = \frac{\lambda^*}{\widehat{\lambda}^*} \frac{\frac{W_A}{W_B}}{\frac{\widehat{W}_A}{\widehat{W}_B}}. \tag{14}$$

We recall that when agents are initially endowed with a fixed fraction of aggregate endowment, $\frac{e_{A,0}}{e_{B,0}} = \frac{e_{A,1}(s)}{e_{B,1}(s)}$ $\forall s \in \mathbb{S}$, then the initial wealth ratios in the two economies with and without early information releases are identical, $\frac{W_A}{W_B} = \frac{\widehat{W}_A}{\widehat{W}_B}$. Equation (14) then implies that in equilibrium the ratio of Pareto weights in the two economies provides us with a clear relation between the respective ratios of agents' expected utilities,

$$\frac{\frac{U_A\left(c^*_A;\mathbb{P}_A\right)}{U_B\left(c^*_B;\mathbb{P}_B\right)}}{\frac{U_A\left(\widehat{c}^*_A;\mathbb{P}_A\right)}{U_B\left(\widehat{c}^*_B;\mathbb{P}_B\right)}} = \frac{\lambda}{\widehat{\lambda}^*}. \tag{15}$$

Next, we turn to an *unambiguous* relation between equilibrium consumption plans and the Pareto weight, which holds in each state and point in time and independent of the availability and nature of information. We differentiate the FOC (34) and the market clearing condition (35) with respect to the Pareto weight for the economy with early information releases at time 1 (note that the following derivation is equivalent for the economy with late uncertainty resolution),

$$\frac{\beta_A^t}{\beta_B^t}\frac{\mathbb{P}_A(s)}{\mathbb{P}_B(s)}\frac{\partial^2 u_A\left(\widehat{c}_{A,t}^*(s)\right)}{\partial \widehat{c}_{A,t}^2(s)}\frac{\partial \widehat{c}_{A,t}^*(s)}{\partial \widehat{\lambda}^*} = \frac{\partial u_B\left(\widehat{c}_{B,t}^*(s)\right)}{\partial \widehat{c}_{B,t}(s)} + \widehat{\lambda}^*\frac{\partial^2 u_B\left(\widehat{c}_{B,t}^*(s)\right)}{\partial \widehat{c}_{B,t}^2(s)}\frac{\partial \widehat{c}_{B,t}^*(s)}{\partial \widehat{\lambda}^*},$$

and

$$\frac{\partial \widehat{c}_{A,1}^*(s)}{\partial \widehat{\lambda}^*} = -\frac{\partial \widehat{c}_{B,1}^*(s)}{\partial \widehat{\lambda}^*} = \frac{1}{\widehat{\lambda}^*}\left(\frac{\frac{\partial^2 u_A\left(\widehat{c}_{A,1}^*(s)\right)}{\partial \widehat{c}_{A,1}^2(s)}}{\frac{\partial u_A\left(\widehat{c}_{A,t}^*(s)\right)}{\partial \widehat{c}_{A,t}(s)}} + \frac{\frac{\partial^2 u_B\left(\widehat{c}_{B,1}^*(s)\right)}{\partial \widehat{c}_{B,1}^2(s)}}{\frac{\partial u_B\left(\widehat{c}_{B,t}^*(s)\right)}{\partial \widehat{c}_{B,t}(s)}}\right)^{-1}, \tag{16}$$

$\forall s \in \mathbb{S}, \forall t \in \{0,1\}$.

Since agents prefer more to less and are risk averse, the above expressions imply that in every state and at any time agent A's (B's) equilibrium consumption always decreases (increases) with the Pareto weight in either economy with or without early information releases. Consequently, within the same economy and keeping aggregate endowment fixed, any change in the Pareto weight will make one agent strictly better and the other strictly worse off. Because our convention for the Pareto weight in the FOC (28), (29) and (34) is such that the Pareto weight characterizes the relative influence of agent B on the equilibrium (compare to that of agent A),[11] it is intuitive that B's consumption increases with the weight while A's consumption decreases by virtue of market clearing.

Going beyond this intuition, the above result assert that in all states and points in time B's (A's) consumption *invariably* increases (decreases) with the Pareto weight. The setting most appropriate to the above relation is where aggregate endowment

[11] In fact, one can show that Pareto weights (λ^* or $\widehat{\lambda}^*$) are increasing in ratio of agent B's initial wealth over agent A's.

is fixed but individual agents' endowments change. We note that up to the Pareto weight, the systems (28) through (33) and (34) through (38) at $t = 1$ are identical. This observation combined with the above relation between equilibrium consumptions and Pareto weights implies the following cross-economy comparative statics,

$$\begin{array}{l} \lambda^* > \widehat{\lambda}^* \Longrightarrow \begin{cases} c^*_{A,1}(s) < \widehat{c}^*_{A,1}(s), & \forall s \in \mathbb{S} \Longrightarrow U_{A,1} < \widehat{U}_{A,1} \\ c^*_{B,1}(s) > \widehat{c}^*_{B,1}(s), & \forall s \in \mathbb{S} \Longrightarrow U_{B,1} > \widehat{U}_{B,1} \end{cases} \\ \lambda^* < \widehat{\lambda}^* \Longrightarrow \begin{cases} c^*_{A,1}(s) > \widehat{c}^*_{A,1}(s), & \forall s \in \mathbb{S} \Longrightarrow U_{A,1} > \widehat{U}_{A,1} \\ c_{B,1}(s) < \widehat{c}_{B,1}(s), & \forall s \in \mathbb{S} \Longrightarrow U_{B,1} < \widehat{U}_{B,1} \end{cases} \end{array} . \tag{17}$$

Though eventually the Pareto weight is an endogenous quantity in an economy, equilibrium consumptions' unambiguous dependence on the Pareto weight proves crucial in establishing the welfare implications of early information releases. Proposition 3 provides the comparative statics results at time 0.

Proposition 3 *(Period $t = 0$-comparative statics) Suppose agents have constant relative risk aversions $\gamma_A \geq 1$, $\gamma_B \geq 1$.*

1. *When $\lambda \geq \widehat{\lambda}$, agent A's expected utility in period $t = 0$ is higher in the economy with early information releases than it is in the economy with late uncertainty resolution, $\widehat{U}_{A,0} > U_{A,0}$.*

2. *When $\widehat{\lambda} \geq \lambda$, agent B's expected utility in period $t = 0$ is higher in the economy with early information releases than it is in the economy with late uncertainty resolution, $\widehat{U}_{B,0} > U_{B,0}$.*

3. *When $\widehat{\lambda} = \lambda$, both agents' expected utilities in period $t = 0$ are higher in the economy with early information releases than it is in the economy with late uncertainty resolution, $\widehat{U}_{A,0} > U_{A,0}$ and $\widehat{U}_{B,0} > U_{B,0}$.*

Key to this result is the convexity of agent $i \in I$'s utility with respect to the Pareto weight in each state at time 0. The above inequalities are a simple applicaiton of Jensen's inequality.

We are now ready to establish the main result on the relevance of the timing of information releases when agents hold heterogeneous beliefs about the prospects of the economy. Early information releases about the future endowment are unambiguously Pareto improving.

Proposition 4 *(Relevance of information) Under the assumptions that (i) markets are complete, (ii) agents are sufficiently risk-averse ($\gamma_A > 1$, $\gamma_B > 1$), and (iii) agents are initially endowed with some portion of the endowment tree, expected utilities for both agents are strictly higher in the economy with early information releases than they are in the economy with late uncertainty resolution,*

$$U_i\left(\widehat{c}_i^*; \mathbb{P}_i\right) > U_i\left(c_i^*; \mathbb{P}_i\right), \ \forall i \in I. \tag{18}$$

Proposition 4 holds for any level of heterogeneity in beliefs, risk aversions, time preferences and initial wealths.

3.2.3 Imperfect Signals/ Two Trees

So far our analysis has focused on the case of perfect signals, the released information perfectly forecasts future endowments. When signals are imperfect, investors can reduce but not completely eliminate the uncertainty in future endowments. We briefly examine the relevance of imperfect signals on risk-sharing and welfare.

We define the filtration $\widetilde{\mathcal{F}}$ with $\widetilde{\mathcal{F}}_{-1} = \mathcal{F}_{-1}$, $\mathcal{F}_0 \subset \widetilde{\mathcal{F}}_0 \subset \widehat{\mathcal{F}}_0$ and $\widetilde{\mathcal{F}}_1 = \mathcal{F}_1$, and the partitions $\widetilde{\mathcal{P}}_t$ consisting of all the atoms of the σ-algebra $\widetilde{\mathcal{F}}_t$, $\forall t \in \{0, 1\}$. At $t = 0^-$ agents observes a perfect signal about $\widetilde{e}_1(k)$ for $k \in \widetilde{\mathcal{P}}_0$ of the total future endowment

$e_1(s) = \widetilde{e}_1(k) + \epsilon(s)$ to be realized at $t = 1$ in state $s \in \mathbb{S}$.[12] We assume that $\widetilde{e}_1(s)$ is constant over $s \in k$, $\forall k \in \widetilde{\mathcal{P}}_0$. $\widetilde{e}_1$ is $\widetilde{\mathcal{F}}_0$ measurable and random variable ϵ is $\widetilde{\mathcal{F}}_1$ measurable. Endowments at time $t = 0$ remain constant and known to all agents as in the previous section.

The imperfect signal setting also can be interpreted as an economy with two trees where one tree produces $\widetilde{e}_1$ and the second tree ϵ at time 1. In such a two trees interpretation of the model agents observe a perfect signal only about one tree, namely the tree producing $\widetilde{e}_1$ at $t = 1$.

The complete market offers agents to trade (at $t = -1$) two sets of Arrow-Debreu securities paying off at $t = 0$ and $t = 1$, respectively. As in section 2 let $q_{0,k}$, $q_{1,s}$ denote the prices of these securities at time -1. The FOC and market clearing conditions associated with each agent's optimization problem are

$$t = 0 \begin{cases} \mathbb{P}_A(k) \frac{\partial u_A(\widetilde{c}^*_{A,0}(k))}{\partial \widetilde{c}_{A,0}(k)} = \widetilde{\lambda}^* \mathbb{P}_B(k) \frac{\partial u_B(\widetilde{c}^*_{B,0}(k))}{\partial \widetilde{c}_{B,0}(k)} \\ \widetilde{c}^*_{A,0}(k) + \widetilde{c}^*_{B,0}(k) = e_0 \end{cases} , \quad \forall k \in \widetilde{\mathcal{P}}_0$$

$$t = 1 \begin{cases} \mathbb{P}_A(s) \beta_A \frac{\partial u_A(\widetilde{c}^*_{A,1}(s))}{\partial \widetilde{c}_{A,1}(s)} = \widetilde{\lambda}^* \mathbb{P}_B(s) \beta_B \frac{\partial u_B(\widetilde{c}^*_{B,1}(s))}{\partial \widetilde{c}_{B,1}(s)} \\ \widetilde{c}^*_{A,1}(s) + \widetilde{c}^*_{B,1}(s) = \widetilde{e}_1(k) + \epsilon(s) \end{cases} , \quad \forall s \in \mathbb{S},$$

and the budget constraint for agent A,[13]

$$\sum_{k \in \widetilde{\mathcal{P}}_0} q^*_{0,k} \left(\widetilde{c}^*_{A,0}(k) - e_{A,0} \right) + \sum_{s \in \mathbb{S}} q^*_{1,s} \left(\widetilde{c}_{A,1}(s) - e_{A,1}(s) \right) = 0.$$

[12] That is, $e_{A,1}(s) + e_{B,1}(s) = e_1(s) = \widetilde{e}_1(k) + \epsilon(s)$ for all $s \in \mathbb{S}$. We assume $\widetilde{e}_1(k) \in \mathbb{R}_+, \forall k \in \widetilde{\mathcal{P}}_0$ and $\epsilon(s) \in \mathbb{R}_+, \forall s \in \mathbb{S}$. The random ratio $\frac{\widetilde{e}_1}{\epsilon}$ can be construed to characterize the economic "informativeness" of the signal.

[13] Budget constraint for B follows automatically and redundantly from the market clearing conditions.

We immediately note that due to the assumption that agents receive the same fraction of aggregate endowment at each point in time and state of the world, only aggregate endowments e_0 and e_1 ($= \widetilde{e}_1 + \epsilon$) affect the equilibrium consumptions. The equilibrium consumption plans have the following functional forms (where $F_{i,t}(x)$ denotes some implicit function of generic arguments x),[14]

$$\begin{aligned}
\widetilde{c}^*_{i,0}(k) &= F_{i,0}\left(\left\{\mathbb{P}_A\left(s\right), \mathbb{P}_B\left(s\right)\right\}_{s\in\mathbb{S}}, e_0, \left\{\widetilde{e}_1(k) + \epsilon(s)\right\}_{k\in\widetilde{\mathcal{P}}_0, s\in\mathbb{S}}\right) \\
\widetilde{c}^*_{i,1}(s) &= F_{i,1}\left(\left\{\mathbb{P}_A\left(s\right), \mathbb{P}_B\left(s\right)\right\}_{s\in\mathbb{S}}, e_0, \left\{\widetilde{e}_1(k) + \epsilon(s)\right\}_{k\in\widetilde{\mathcal{P}}_0, s\in\mathbb{S}}\right),
\end{aligned}$$

$\forall i \in I, k \in \widetilde{\mathcal{P}}_0, s \in \mathbb{S}$.

There are two important conclusions we draw from the above equations. First, when agents have homogeneous beliefs, $\mathbb{P}_A \equiv \mathbb{P}_B$, the beliefs cancel out in the FOC at both $t = 0$ and $t = 1$. Consequently, the equilibrium consumption allocation at time 0 is state-independent. Following the argument leading to Proposition 1, we conclude that observing an imperfect signal does neither improve nor diminish welfare. In other words, Proposition 1 remains valid when early information releases are imperfect.

Second, when agents have heterogeneous beliefs, $\mathbb{P}_A\left(s\right) \neq \mathbb{P}_B\left(s\right)$ for some $s \in k, k \in \widetilde{\mathcal{P}}_0$, it is the information structure of the economy and the heterogeneity in beliefs but not the 'informativeness" ratio $\frac{\widetilde{e}_1}{\epsilon}$ which matters for early information releases to be Pareto improving. To see this, we keep a specific information and (heterogeneous) beliefs structure $\widetilde{\mathcal{F}}$ and $\{\mathbb{P}_A, \mathbb{P}_B\}$ fixed and only vary the ratio $\frac{\widetilde{e}_1}{\epsilon}$ across states $s \in \mathbb{S}$. The aggregate endowment vector $e_1 = \widetilde{e}_1 + \epsilon$ stays unchanged in our comparative statics.[15] The equilibrium consumption allocation and expected

[14] To see this, note that in the above budget constraint for agent A, AD prices are $q^*_{0,k} = \mathbb{P}_A(k)\frac{\partial u_A(\widetilde{c}^*_{A,0}(s_0))}{\partial \widetilde{c}_{A,0}(s_0)}$, $q^*_{1,s} = \mathbb{P}_A(s)\beta_A\frac{\partial u_A(\widetilde{c}^*_{A,1}(s))}{\partial \widetilde{c}_{A,1}(s)}$.

[15] Changing aggregate endowment will affect the equilibrium consumption allocation but this

utilities for all agents stay unchanged. Accordingly, the ratio $\frac{\widetilde{e}_1}{\epsilon}$ does not have any welfare implications. In contrast, a change in the information structure $\widetilde{\mathcal{F}}$ and heterogeneity in beliefs $\{\mathbb{P}_A, \mathbb{P}_B\}$ crucially affect the equilibrium consumption allocation and welfare (Proposition 4).

Notice that early information relesaes are Pareto improving according to Proposition 4 even in the case where $\widetilde{e}_1(k) = 0, \forall k \in \widetilde{\mathcal{P}}_0$, that is, even if the signal observed at time 0 is unrelated to aggregate endowment (information about idiosyncratic event). This insight reinforces the intuition that in a heterogeneous beliefs setting early information releases are Pareto improving due to speculation.

Interpreting the model as an economy with two trees, we conclude that early information releases about a tiny stock are as valuable as signals about a large tree since the ratio $\frac{\widetilde{e}_1}{\epsilon}$ is not of importance when determining equilibrium allocations and expected utilities. However, the information structure and the degree of heterogeneity in beliefs matters when comparing the welfare implications of early information releases about two different stocks.

Intuitively, the size of a tree do not matter because early information releases improve welfare due to disagreements about the "fundamental value" of state contingent securities and speculation. It does not matter whether agents disagree about the outcome of future aggregate endowment or an idiosyncratic event. Disagreement induces in either case an opportunity to speculate and increase ex-ante expected utilities of all agents.

---

supply change is not due to the information channel we focus on in this paper. For comparative statics we keep the vector $e_1 = \widetilde{e}_1 + \epsilon$ fixed throughout.

3.2.4 Numerical Illustration

A numerical example is useful to further illustrate our qualitative results and get a sense of the quantitative importance. Moreover, the asset pricing implications of the timing of information releases (equations (1), (2), (3)) are analytically intractable. Numerical solutions allow us to get a sense of how pricing differs between economies with early versus late uncertainty resolution.

For our numerical exercise we focus on the case of power utility, $u_i(x) = \frac{x^{1-\gamma_i}}{1-\gamma_i}$ in equaiton (6). We choose the state space to be $\mathbb{S} = \{low, high\}$ with $e_0 = 1$, $e_1(high) = e^{\frac{0.03}{12}+0.08\sqrt{\frac{1}{12}}}$ and $e_1(low) = e^{\frac{0.03}{12}-0.08\sqrt{\frac{1}{12}}}$. Given $\mathbb{P}_B(high) = 0.5$ and setting one time period equal to 1 month, the parameters imply that under agent B's belief aggregate dividends are expected to grow by roughly 3% per year and have a annual volatility of 8%. This assumption is in line with other calibrations in the literature (e.g. Garleanu and Pedersen, 2011). A monthly time interval seems reasonable if we think about for instance the difference between dividend announcement date and actual payout date. We choose agents to be homogeneous with respect to their time preferences, relative risk aversions and initial endowments and set $\beta_A = \beta_B = 0.9996$, $\gamma_A = \gamma_B = 2$ and $\frac{e_A}{e} = 0.5$. The only heterogeneity between agents will be the probability they assign to the *high* state. We fix $\mathbb{P}_B(high) = 0.5$ (agent B beliefs *low* and *high* states are equally likely to realize) and compute equilibria in the two economies with and without early information releases for all $\mathbb{P}_A(high) \in [0.1, 0.9]$.

Consistent with our qualitative results figure 2 illustrates that early releases of public information improve risk-sharing. Ex-ante utility of both agents is larger in the case of early information releases than in the benchmark case of late uncertainty resolution. The red line with circle markers plots $\frac{u_i^{-1}\left(U_i\left(\widehat{c}_i^*;\mathbb{P}_i\right)\right)}{u_i^{-1}\left(U_i\left(c_i^*;\mathbb{P}_i\right)\right)} - 1$ against $\mathbb{P}_A(high)$,

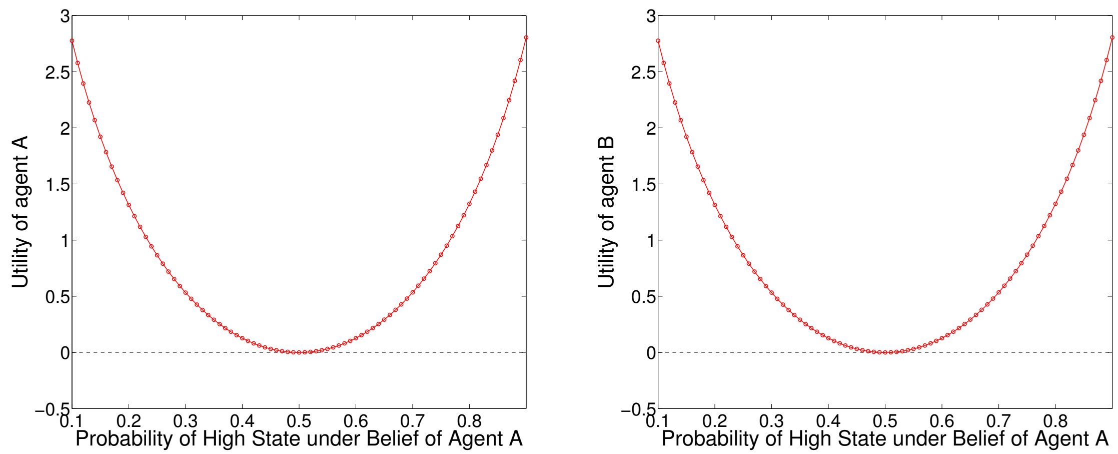


Figure 2: The horizontal axis measures the probability agent A assigns to the *high* state, $\mathbb{P}_A\,(high)$. The red line with circle markers plots $\frac{u_i^{-1}\left(U_i\left(\widehat{c}_i^*;\mathbb{P}_i\right)\right)}{u_i^{-1}\left(U_i\left(c_i^*;\mathbb{P}_i\right)\right)} - 1$, the percentage of additional lifetime consumption agent $i \in I$ requires in the economy with late uncertainty resolution in order to be indifferent to the equilibrium consumption allocation he would receive in the economy with early information releases. The vertical axis is measured in percentage points. The left panel illustrates the case of $i = A$ and the right panel $i = B$. For both agents utility is larger in the economy with early information releases than in the economy with late uncertainty resolution.

where $u_i^{-1}(x) = (1-\gamma_i)\, x^{\frac{1}{1-\gamma_i}}$ is the inverse fucntion of $u_i(x)$.[16] That is, the red line with circle markers measures what percentage of lifetime consumption agent $i \in I$ was willing to give up in order to live in an economy with early uncertainty resolution instead of a world with no public information releases. The numerical solutions also confirms that the early information releases are particularly valuable in the case of large disagreements between agents.

Figure 3 illustrates that the ex-ante equity premium of a dividend strip (equation (3)) is essentially unaffected by the timing of public information releases. The black solid line plots the annualized equity premium in the economy with late uncertainty resolution against $\mathbb{P}_A(high)$, the red line with circle markers the annualized premium in the economy with early information releases. Black solid and red line with circle markers are numerically indistinguishable. The left panel plots the equity premium under agent A's belief and the right panel under B's belief.

Comparative statics analyses help to further explore our results. An increase in agent A's relative endowment $\frac{e_A}{e}$ causes $\frac{U_A\left(\widehat{c}_A^*;\mathbb{P}_A\right)}{U_A\left(c_A^*;\mathbb{P}_A\right)}$ to decrease and $\frac{U_B\left(\widehat{c}_B^*;\mathbb{P}_B\right)}{U_B\left(c_B^*;\mathbb{P}_B\right)}$ to increase. Intuitively, the relatively poor agent profits more from an early release of information and speculation because his absolute risk aversion (ARA) is larger than the ARA of the relatively rich agent. In equilibrium a speculative bet always has to be in favor of the party with the larger ARA. The willingness to participate in speculation is decreasing in risk aversion and a relatively more risk averse agent need to perceive more convincing odds to participate in speculation than a relatively less risk averse agent. The equity premium does not essentially differ between an economy with or without early information releases.

[16]Note that $\frac{u_i^{-1}(U_i(\widehat{c}_i^*;\mathbb{P}_i))}{u_i^{-1}\left(U_i\left(c_i^*;\mathbb{P}_i\right)\right)} - 1 = x$ implies $u_i^{-1}\left(U_i\left(\widehat{c}_i^*;\mathbb{P}_i\right)\right) = u_i^{-1}\left(U_i\left((1+x)\, c_i^*;\mathbb{P}_i\right)\right)$, agent $i \in I$ values an early release of public information with a fraction x of his lifetime consumption.

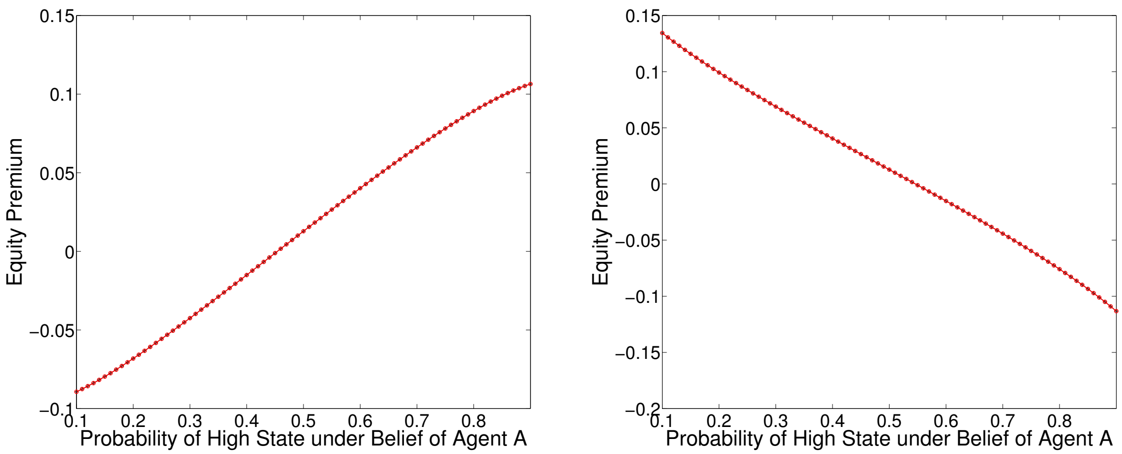


Figure 3: The horizontal axis measures the probability agent A assigns to the *high* state, $\mathbb{P}_A(high)$. The vertical axis measures the annualized equity premium of a dividend strip (equation (3)) which pays aggregate endowment at time 1. The black solid line is the equity premium in the economy with late uncertainty resolution, the red line with circle markers the premium in the economy with early information releases (black solid and red line with circle markers are numerically indistinguishable). The left panel plots the equity premium under agent A's belief and the tight panel under B's belief.

In the same spirit, we find that $\frac{U_i\left(\widehat{c}_i^*;\mathbb{P}_i\right)}{U_i\left(c_i^*;\mathbb{P}_i\right)}$ decreases if we increase the RRA (or ARA) of both agents, and $\frac{U_A\left(\widehat{c}_A^*;\mathbb{P}_A\right)}{U_A\left(c_A^*;\mathbb{P}_A\right)}$ decreases more than $\frac{U_B\left(\widehat{c}_B^*;\mathbb{P}_B\right)}{U_B\left(c_B^*;\mathbb{P}_B\right)}$ as we increase γ_A and keep γ_B constant. The equity premium is increasing in RRA but again it does not differ across two economies.

3.2.5 Public Information and Incomplete Markets

In his seminal paper Hirshleifer (1971) illustrates with a numerical example that public information may decrease agents' expected utilities if a signal is released before they have time to trade and share risk. We re-examine Hirshleifer's (1971) example but add heterogeneity in beliefs to the model. In other words, we adapt our model of early information releases (economy with filtration $\widehat{\mathcal{F}}$) by introducing incomplete markets and do not allow any trade before time 0. The time line of our incomplete market model is illustrated in Figure 4.

In comparison to the economies described in Figure 1, agents cannot insure each other and smooth consumption across the state space $\mathbb{S}$ but they can trade after the state of the world is revealed and smooth consumption between time 0 and 1. Given state $s \in \mathbb{S}$ realizes at time 0 agent $i \in I$ maximizes

$$\sup_{\left\{\widehat{\widehat{c}}_i(s)\in\mathbb{R}_+^2\right\}} \left\{ \sum_{t\in\{0,1\}} \beta_i^t u_i\left(\widehat{\widehat{c}}_{i,t}\left(s\right)\right) \right\} \;\; s.t. \;\; 0 \geq \widehat{\widehat{c}}_{i,0}\left(s\right) - e_{i,0} + \frac{\widehat{\widehat{c}}_{i,1}\left(s\right) - e_{i,1}\left(s\right)}{\widehat{\widehat{R}}\left(s\right)}, \quad (19)$$

where $\widehat{\widehat{R}}\left(s\right)$ is the gross risk-free interest rate between time 0 and 1 in state $s \in \mathbb{S}$. Market clearing and equilibrium are defined as in Section 2.4. The equilibrium quantities $\widehat{\widehat{c}}_A^*\left(s\right)$, $\widehat{\widehat{c}}_B^*\left(s\right)$, $\widehat{\widehat{R}}^*\left(s\right)$ and $\widehat{\widehat{\lambda}}_i^*\left(s\right)$ satisfy the system of equations (39) through (43) in the Appendix.

In the special case of log-utility we can derive explicit solutions (see Appendix).

No trade before a Perfect Signal/ Incomplete Markets:

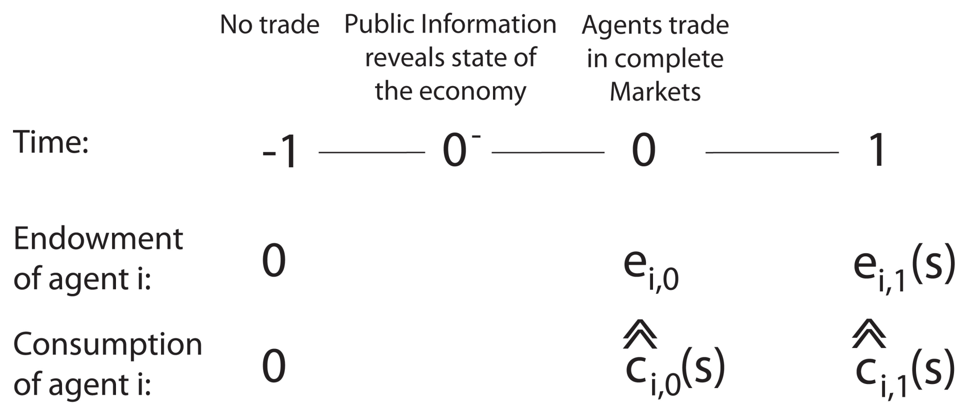


Figure 4: Incomplete market economy with an early release of a perfect public signal or early uncertianty resolution. Agents are not allowed to trade before the release of a perfect signal.

Agent $i \in I$'s expected utility at time -1 is

$$
\begin{aligned}
U_i\left(\widehat{\widehat{c}}_i^{*};\mathbb{P}_i\right) &= \ln\left(e_0\right) + E^i\left[\beta_i \ln\left(e_t\left(s\right)\right)\right] - \ln\left(1+\frac{e_{j,0}\left(1+\beta_i\right)}{e_{i,0}\left(1+\beta_j\right)}\right) \\
&\quad - \beta_i \ln\left(1+\frac{e_{j,0}\beta_j\left(1+\beta_i\right)}{e_{i,0}\beta_i\left(1+\beta_j\right)}\right), \quad \forall i,j \in I, i \neq j.
\end{aligned}
$$

$U_i\left(\widehat{\widehat{c}}_i^{*};\mathbb{P}_i\right) \leq U_i\left(c_i^*;\mathbb{P}_i\right)$ holds by Jensen's inequality,

$$
\begin{aligned}
\beta_i E^i\left[\ln\left(1+\frac{e_{j,0}\beta_j\left(1+\beta_i\right)\mathbb{P}_j\left(s\right)}{e_{i,0}\beta_i\left(1+\beta_j\right)\mathbb{P}_i\left(s\right)}\right)\right] &\leq \beta_i \ln\left(E^i\left[1+\frac{e_{j,0}\beta_j\left(1+\beta_i\right)\mathbb{P}_j\left(s\right)}{e_{i,0}\beta_i\left(1+\beta_j\right)\mathbb{P}_i\left(s\right)}\right]\right) \\
&= \beta_i \ln\left(1+\frac{e_{j,0}\beta_j\left(1+\beta_i\right)}{e_{i,0}\beta_i\left(1+\beta_j\right)}\right).
\end{aligned}
$$

In incomplete markets an early release of information makes all agents always ex-ante worse off because the state of the world is revealed before trading takes place and agents are not able to speculate on their beliefs. In this special case all agents have the same RRA and endowments are proportional, which implies that if there was no heterogeneity in beliefs, then agents' initial endowments equal the equilibrium consumption allocation and the market incompleteness did not matter. In particular, if beliefs were homogeneous, then $\widehat{\widehat{c}}_i^* = c_i^*$ and $U_i\left(\widehat{\widehat{c}}_i^*; \mathbb{P}_i\right) = U_i\left(c_i^*; \mathbb{P}_i\right)$. In other words, in this special case it is not the well-known Hirshleifer (1971) effect that makes agents worse off but in incomplete markets early uncertainty resolution reduces expected profits from speculation.

We numerical solve our model in a more general case of power utility $u_i(x) = \frac{x^{1-\gamma_i}}{1-\gamma_i}$ and show that early information releases may increase expected utility of all agents in an incomplete market setting. As in the earlier numerical example we choose the state space to be $\mathbb{S} = \{low, high\}$ with $e_0 = 1$, $e_1(high) = e^{\frac{0.03}{12}+0.08\sqrt{\frac{1}{12}}}$ and $e_1(low) = e^{\frac{0.03}{12}-0.08\sqrt{\frac{1}{12}}}$. Agents are homogeneous with respect to their time preference and initial endowment and we set $\beta_A = \beta_B = 0.9996$ and $\frac{e_A}{e} = 0.5$. Agents differ with respect to RRA, $\gamma_A = 30$ and $\gamma_B = 2$. We fix $\mathbb{P}_B(high) = 0.5$ (agent B beliefs *low* and *high* states are equally likely to realize) and compute equilibria in the three economies described in Figures 1 and 4 for all $\mathbb{P}_A(high) \in [0.1, 0.9]$.

Figure 5 illustrates that in incomplete markets early information releases may be Pareto improving if beliefs are heterogeneous. The red line with circle markers plots $\frac{u_i^{-1}\left(U_i\left(\widehat{c}_i^*;\mathbb{P}_i\right)\right)}{u_i^{-1}\left(U_i\left(c_i^*;\mathbb{P}_i\right)\right)} - 1$, the green solid line $\frac{u_i^{-1}\left(U_i\left(\widehat{\widehat{c}}_i^*;\mathbb{P}_i\right)\right)}{u_i^{-1}\left(U_i\left(c_i^*;\mathbb{P}_i\right)\right)} - 1$. In the cases of $\mathbb{P}_B(high) = 0.5$ and $\mathbb{P}_A(high) \in [0.6, 0.87]$ all agents are better off in an economy with early information releases but no trade before time 0 than in an economy with late uncertainty resolution. Our result makes an important contribution to the Hirshleifer's (1971) findings. Hirshleifer (1971) illustrates that an early release of information may reduce risk-sharing and has negative effects on agents' expected utilities. His results appear to hold in the case of homogeneous beliefs but we show that it may reverse in a heterogeneous beliefs setting. Our result appears to hold for any $\gamma_A \neq \gamma_B$. However,

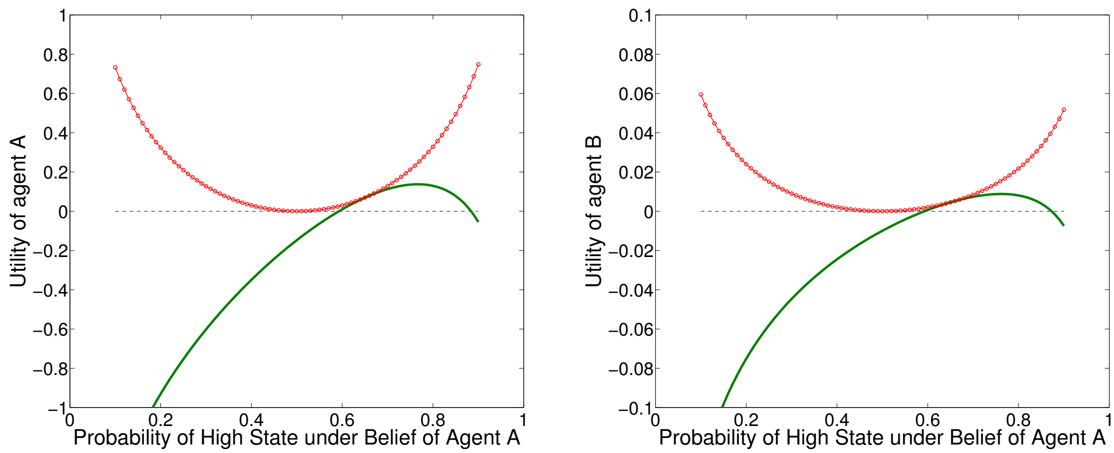


Figure 5: The horizontal axis measures the probability agent A assigns to the *high* state, $\mathbb{P}_A(high)$. The red line with circle markers plots $\frac{u_i^{-1}\left(U_i\left(\widehat{c}_i^*;\mathbb{P}_i\right)\right)}{u_i^{-1}\left(U_i\left(c_i^*;\mathbb{P}_i\right)\right)}-1$, the percentage of additional lifetime consumption agent $i \in I$ requires in the economy with late uncertainty resolution in order to be indifferent to the equilibrium consumption allocation he would receive in the economy with early information releases and complete markets (trading is possible before uncertainty resolution at time -1). The green solid line plots $\frac{u_i^{-1}\left(U_i\left(\widehat{\widehat{c}}_i^*;\mathbb{P}_i\right)\right)}{u_i^{-1}\left(U_i\left(c_i^*;\mathbb{P}_i\right)\right)}-1$, the percentage of additional lifetime consumption agent $i \in I$ requires in the economy with late uncertainty resolution in order to be indifferent to the equilibrium consumption allocation he would receive in the economy with early infromation releases and incomplete markets (trading is only possible after risk is resolved at time 0). The vertical axis is measured in percentage points. The left panel illustrates the case of $i = A$ and right panel $i = B$.

choosing a large spread between agents' RRAs increases the region over $\mathbb{P}_A\,(high)$ for which $U_i\left(\widehat{c}_i^*;\mathbb{P}_i\right) > U_i\left(c_i^*;\mathbb{P}_i\right)$.

Less surprising is the result that all agents always prefer to live in a world with early information releases and access to complete markets at time -1.

4 Recursive Preferences

4.1 Optimal Risk-Sharing and Pricing

We turn now to the more general case of recursive preferences. We employ the specification introduced by Epstein and Zin (1989), which incorporates preferences over the timing of uncertainty resolution (Kreps and Porteus, 1978),[17]

$$U_i\left(c_i;\mathbb{P}_i,\mathcal{F}\right) = \left(E^i\left[\left(c_{i,0}^{\rho_i} + \beta_i\left(E^i\left[\left.c_{i,1}^{1-\gamma_i}\right|\mathcal{F}_0\right]\right)^{\frac{\rho_i}{1-\gamma_i}}\right)^{\frac{1-\gamma_i}{\rho_i}}\middle|\,\mathcal{F}_{-1}\right]\right)^{\frac{1}{1-\gamma_i}}.$$

The EIS is given by $\frac{1}{1-\rho_i}$ and γ_i controls for RRA of agent $i \in I$. The special case of power utility is recovered for $\gamma_i = 1-\rho_i$. By construction agent $i \in I$ has a preference for early (late) uncertainty resolution if $\gamma_i > (<)\, 1-\rho_i$.

In the economy with no early information releases (filtration $\mathcal{F}$) the utility function of agent $i \in I$ can be simplified to

$$U_i\left(c_i;\mathbb{P}_i,\mathcal{F}\right) = \left(c_{i,0}^{\rho_i} + \beta_i\left(E^i\left[\left.\left(c_{i,1}\left(s\right)\right)^{1-\gamma_i}\right|\mathcal{F}_0\right]\right)^{\frac{\rho_i}{1-\gamma_i}}\right)^{\frac{1}{\rho_i}}. \tag{20}$$

If a perfect signal resolves all uncertainty already at time 0 (filtration $\widehat{\mathcal{F}}$), the utility function of agent $i \in I$ becomes

$$U_i\left(\widehat{c}_i;\mathbb{P}_i,\widehat{\mathcal{F}}\right) = \left(E^i\left[\left(\left(\widehat{c}_{i,0}\left(s\right)\right)^{\rho_i} + \beta_i\left(\widehat{c}_{i,1}\left(s\right)\right)^{\rho_i}\right)^{\frac{1-\gamma_i}{\rho_i}}\middle|\,\widehat{\mathcal{F}}_{-1}\right]\right)^{\frac{1}{1-\gamma_i}}. \tag{21}$$

[17]Notice that similar results are achieved with other recursive utility functions such as internal-habit preferences, which do not assume an inherent preference over the timing of uncertainty resolution.

$\frac{U_i\left(\widehat{c}_i;\mathbb{P}_i,\widehat{\mathcal{F}}\right)}{U_i(c_i;\mathbb{P}_i,\mathcal{F})} - 1 = x$ implies $U_i\left(\widehat{c}_i;\mathbb{P}_i,\widehat{\mathcal{F}}\right) = U_i\left(\left(1+x\right)c_i;\mathbb{P}_i,\mathcal{F}\right)$, that is, agent $i \in I$ is indifferent between an economy with early information releases and consumption plan $\widehat{c}_i$ or an economy with late uncertainty resolution and consumption allocation $\left(1+x\right)c_i$.

The equilibrium quantities c_A^*, c_B^*, q^* and λ^* in the economy with no early information releases (filtration $\mathcal{F}$) satisfy the system of equations (44) through (49) in the Appendix. $\widehat{c}_A^*$, $\widehat{c}_B^*$, $\widehat{q}^*$ and $\widehat{\lambda}^*$ in the economy with trade before a perfect signal (filtration $\widehat{\mathcal{F}}$) satisfy the system of equations (50) through (54) in the Appendix.

In the special case of homogeneity in beliefs and power utility the equilibrium conditions of the two economies are identical. In contrast, in the general case of $\gamma_i \neq 1-\rho_i$ for some $i \in I$ the equilibria in the two economies are very different from each other. In the general case it is difficult to compare the two economies analytically. We solve our model numerically and comparative statics analyses help us to get an understanding of the differences between the two economies.

The state space in our numerical exercise is $\mathbb{S} = \{low, high\}$ with $e_0 = 1$, $e_1\left(high\right) = e^{\frac{0.03}{12}+0.08\sqrt{\frac{1}{12}}}$ and $e_1\left(low\right) = e^{\frac{0.03}{12}-0.08\sqrt{\frac{1}{12}}}$. We choose agents to be homogeneous with respect to their time preferences, relative risk aversions and beliefs and set $\beta_A = \beta_B = 0.9996$, $\gamma_A = \gamma_B = 10$, $\mathbb{P}_A\left(high\right) = \mathbb{P}_B\left(high\right) = 0.5$. We allocate more initial endowment to agent B, $\frac{e_A}{e} = 0.2$. The qualitative restuls are not affected by the choice of the initial endowment, but it matters for the quantitative size of the results (see discussion below). The crucial difference between agents is with respect to their preferences for consumption smoothing over time. Agent B is assumed to have power utility, $\frac{1}{EIS_B} = 1-\rho_B = \gamma_B$. Agent A has more general recursive preferences and we solve equilibria for all cases of $EIS_A = \frac{1}{1-\rho_A} \in (0,2]$.

Figure 6 illustrates that an early release of public information improves risk-

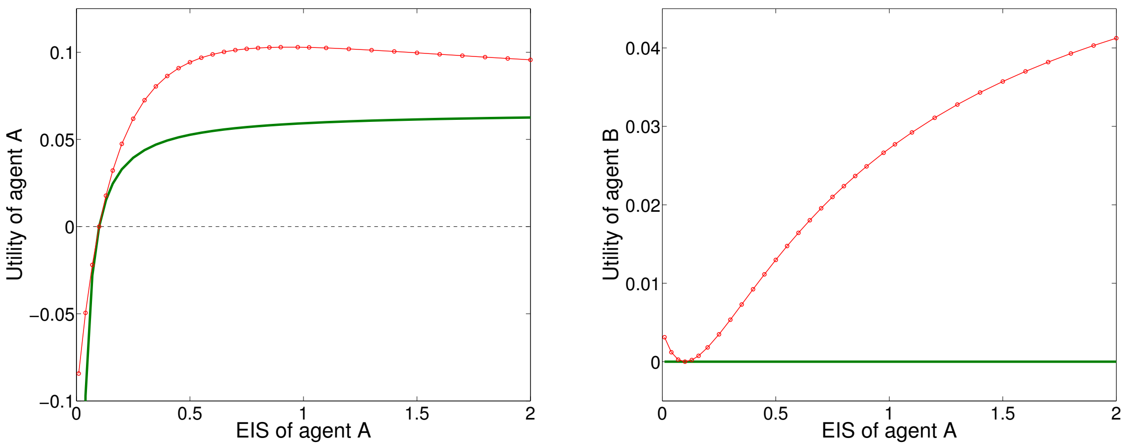


Figure 6: The horizontal axis measures the EIS of agent A. The red line with circle markers plots $\frac{U_i\left(\widehat{c}_i^*;\mathbb{P}_i,\widehat{\mathcal{F}}\right)}{U_i\left(c_i^*;\mathbb{P}_i,\mathcal{F}\right)}-1$, the percentage of additional lifetime consumption agent $i \in I$ requires in the economy with late uncertainty resolution in order to be indifferent to the equilibrium consumption allocation he would receive in the economy with early information releases. The green solid line plots $\frac{U_i\left(c_i^*;\mathbb{P}_i,\widehat{\mathcal{F}}\right)}{U_i\left(c_i^*;\mathbb{P}_i,\mathcal{F}\right)}-1$, the percentage of additional lifetime consumption agent $i \in I$ requires in the economy with late uncertainty resolution in order to be indifferent to a hypothetical situation where risk is resolved early (at time 0) but agents are stuck with the consumption plan c_i^*. The vertical axis is measured in percentage points. The left panel illustrates the case of $i = A$ and right panel $i = B$.

sharing. The red line with circle markers plots $\frac{U_i\left(\widehat{c}_i^*;\mathbb{P}_i,\widehat{\mathcal{F}}\right)}{U_i\left(c_i^*;\mathbb{P}_i,\mathcal{F}\right)} - 1$, the percentage of additional lifetime consumption agent $i \in I$ requires in the economy with late uncertainty resolution in order to be indifferent to the equilibrium consumption allocation he would receive in the economy with early information releases. The green solid line plots $\frac{U_i\left(c_i^*;\mathbb{P}_i,\widehat{\mathcal{F}}\right)}{U_i\left(c_i^*;\mathbb{P}_i,\mathcal{F}\right)} - 1$, the percentage of additional lifetime consumption agent $i \in I$ requires in the economy with late uncertainty resolution in order to be indifferent to a hypothetical situation where the state of the world is revealed at time 0 (early uncertainty resolution) but agents are stuck with the consumption plan c_i^*, the equilibrium consumption allocation in the benchmark economy without early information releases. In a sense, the green solid line caputres the increase in utility due to preferences over the timing of uncertainty resolution, while the difference between the red dotted and green solid line indicates the increase in utility due to risk-sharing.

Agent A prefers late resolution of uncertainty when $EIS_A < \frac{1}{\gamma_A}$ and the green solid line in the left panel of Figure 6 is below zero. He prefers early uncertainty resolution when $EIS_A > \frac{1}{\gamma_A}$ and the green solid line is above zero. At the point $EIS_A = \frac{1}{\gamma_A}$ the green solid line is exactly zero because power utility implies indifference about the timing of uncertainty resolution. Agent B has power utility and does not care about the timing of uncertainty resolution (green solid line is zero for any value of EIS_A in the right panel of Figure 6).

The red line with circle markers is always above the green solid line, implying that early information releases improve risk-sharing. Intuitively, in the case of $EIS_A < (>)\, EIS_B$, agent A is relatively more (less) concerned about consumption smoothing over time than agent B. Agent B (A) will help agent A (B) to smooth his comsumption over time and in return requires a premium for his service. This Pareto improving trade is particularly beneficial if the state of the world is observed at time 0. In the benchmark case without early information releases, consumption at time 1 is uncertain at time 0 and agents benefit relatively less from consumption smoothing.

Figure 6 illustrates that for $EIS_A < \frac{1}{\gamma_A}$ agent A's inherent preference for late uncertainty resolution dominates the benefits of risk-sharing and agent A prefers that no information is released at time 0. In contrast, agent B always prefers early information releases because fo teh risk-sharing benefits. In frictionless markets Coase Theorem implies that a Pareto optimal allocation will be achieved, but it is unclear whether an early release of information is Pareto optimal in general.

In contrast to the discussion on time additive preferences and heterogeneity in beliefs, early information releases are not only Pareto improving (if $EIS_A > \frac{1}{\gamma_A}$) but also welfare improving according to the stricter criterion of Kim (1012), Brunnermeier et al. (2013) and Gilboa et al. (2013).

In the case of recursive preferences an early release of information has substantial implications for pricing. Figure 7 illustrates that for $EIS_A < (>)\, EIS_B$ the ex-ante equity premium of a dividend strip (equation (3)) is substantially higher (lower) in an economy where information is released early than in the benchmark case of late uncertainty resolution. The black solid line at 6.3% is the equity premium in the benchmark economy with late uncertainty resolution. The red line with circle markers is the premium in the economy with information releases at time 0.

The intuition for the result becomes clear if we focus on the pricing kernel implied by agent B's preferences. In the case of power utility the pricing kernel takes a simple form, $\frac{q^*_{1,s}}{\mathbb{P}_B(s)} = \frac{\beta_B \left(c^*_{B,1}(s)\right)^{-\gamma_B}}{E^B\left[\left(c^*_{B,0}(s)\right)^{-\gamma_B}\right]}$, which only depends on the consumption allocation c^*_B but not on the timing of uncertainty resolution. We have seen that for $EIS_A < (>)\, EIS_B$ agent B (A) will help agent A (B) to smooth his comsumption over time, which implies that agent B's consumption plan $\widehat{c}^*_{B,1}(s)$ in the case of early

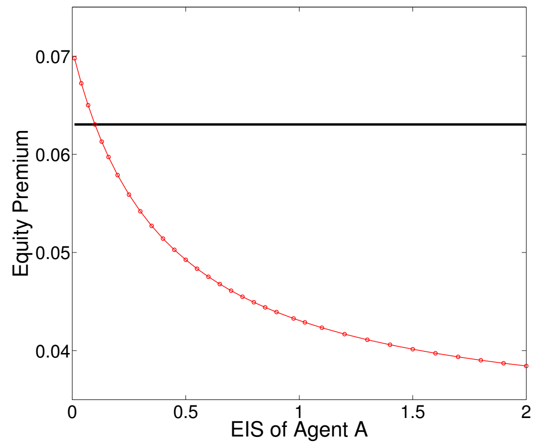


Figure 7: The horizontal axis measures the EIS of agent A. The vertical axis measures the annualized equity premium of a dividend strip (equation (3)) which pays aggregate endowment at time 1. The black solid line is the equity premium in the economy with late uncertainty resolution, the red line with circle markers the premium in the economy with early information releases.

uncertainty resolution is more (less) volatile across states $s \in \mathbb{S}$ than his consumption plan $c^*_{B,1}(s)$ in the benchmark case of late uncertainty resolution. Accordingly, for $EIS_A < (>) \, EIS_B$ the pricing kernel $\frac{q^*_{1,s}}{\mathbb{P}_B(s)}$ covaries stronger (weaker) negatively with endowment $e_1(s)$ and the equity premium is larger (smaller) in the case of early information releases than in the benchmark case of late uncertainty resolution.

The qualitative results of our numerical example appear to hold for a wide range of parameter choices. Crucial appears that the EIS of both agents is “large” enough for an early release of information to be Pareto improving, where “large” in many cases does not require an EIS-value larger than 1. In contrast, the qualitative results about the equity premium appear robust independent of the EIS.

Comparative statics analyses reveal that an increase in the endowment of agent

A, $\frac{e_A}{e}$ is associated with a smaller (larger) change in agent A's (B's) utility and a larger change in the equity premium as we switch from an economy with late to early uncertainty resolution.

Intuitively, suppose the relatively poor agent cares relatively more about consumption smoothing over time than the relatively rich agent. The rich agent has to increase volatility in his consumption plan only a little bit in order to help the poor agent to smooth consumption. In equilibrium the poor agent has to pay a relatively small premium to compensate the rich agent. Accordingly, the poor agent benefits more from risk-sharing due to an early information release than the rich agent. A similar intuition applies to the case when the relatively poor agent cares relatively less about the consumption smoothing over time than the relatively rich agent.

An increase in RRA of both agents, causes the difference in utilities of both agents and the differences in equity premia to be larger between the two economies with late and early uncertainty resolution. Moreover, if the time period in the model is longer (e.g. 1 year instead of 1 month) or aggregate uncertainty is larger, then changes in utilities of both agents appear to be larger as we switch from late to early uncertainty resolution. The difference in equity premia between the two economies appears quantitatively unaffected by changes in the length of the time period or aggregate uncertainty.

4.2 Two Trees

We explore the impact of the size of a stock on our results in Section 4.1. We suppose there are two stocks, a large stock producing endowment e_L and a tiny stock with e_T. Let $\mathcal{F}^{(L)}$ respectively $\mathcal{F}^{(T)}$ be the filtration in an economy where a perfect signal about the large respectively tiny stock is released just before time

0. We define the two economies with filtrations $\mathcal{F}^{(L)}$ respectively $\mathcal{F}^{(T)}$ analogously to the economy described in section 3.2.3, that is, we choose $e_{L,1}\equiv\widetilde{e}_1$ and $e_{T,1} \equiv \epsilon$ respectively $e_{T,1}\equiv\widetilde{e}_1$ and $e_{L,1} \equiv \epsilon$ in the case of $\mathcal{F}^{(L)}$ respectively $\mathcal{F}^{(T)}$. We denote all quantities associated with filtration $\mathcal{F}^{(h)}$ by a superscript $^{(h)}$, $\forall h \in \{L, T\}$. The utility of agent $i \in I$ in the economy with $\mathcal{F}^{(h)}$ $\forall h \in \{L, T\}$ is

$$U_i\left(c_i^{(h)}; \mathbb{P}_i, \mathcal{F}^{(h)}\right) = \left(E^i \left[\left(\begin{array}{c} \left(c_{i,0}^{(h)}(k)\right)^{\rho_i} \\ +\beta_i \left(E^i \left[\left(c_{i,1}^{(h)}(s) \right)^{1-\gamma_i} \middle| \mathcal{F}_0^{(h)} \right] \right)^{\frac{\rho_i}{1-\gamma_i}} \end{array} \right)^{\frac{1-\gamma_i}{\rho_i}} \middle| \mathcal{F}_{-1}^{(h)} \right] \right)^{\frac{1}{1-\gamma_i}} . \tag{22}$$

The equilibrium in an economy with no information releases at time 0 is as described in the previous section (equations (44) through (49) in the Appendix). The equilibrium $\left\{c_A^{(h)*}, c_B^{(h)*}, q^{(h)*}, \lambda^{(h)*}\right\}$ in the economy with early information releases about the large respectively tiny stock is determined in the system of equations (55) through (61) in the Appendix.

In general it is difficult to compare the two economies with $\mathcal{F}^{(T)}$ and $\mathcal{F}^{(T)}$ analytically. We solve our model numerically to get an understanding of the differences between the two worlds. The state space in our numerical exercise is $\mathbb{S} = \{\{low, low\}, \{low, high\}, \{high, low\}, \{high, high\}\}$ where the first entry indicates the state of the tiny stock and the second entry the state of the large stock. We set $e_{L,0} = \frac{2}{3}$, $e_{T,0} = \frac{1}{3}$, $e_{L,1}(low, high) = e_{L,1}(high, high) = \frac{2}{3}e^{\frac{0.03}{12}+0.08\sqrt{\frac{1}{12}}}$, $e_{L,1}(low, low) = e_{L,1}(high, low) = \frac{2}{3}e^{\frac{0.03}{12}-0.08\sqrt{\frac{1}{12}}}$, $e_{T,1}(high, low) = e_{L,1}(high, high) = \frac{1}{3}e^{\frac{0.03}{12}+0.08\sqrt{\frac{1}{12}}}$ and $e_{T,1}(low, low) = e_{L,1}(low, high) = \frac{1}{3}e^{\frac{0.03}{12}-0.08\sqrt{\frac{1}{12}}}$. We choose agents to be homogeneous with respect to their time preferences, relative risk aversions and beliefs and set $\beta_A = \beta_B = 0.9996$, $\gamma_A = \gamma_B = 10$, $\mathbb{P}_A(s) = \mathbb{P}_B(s) = 0.25$ $\forall s \in \mathbb{S}$. As in the previous section, for the initial endowment we choose $\frac{e_A}{e} = 0.2$,

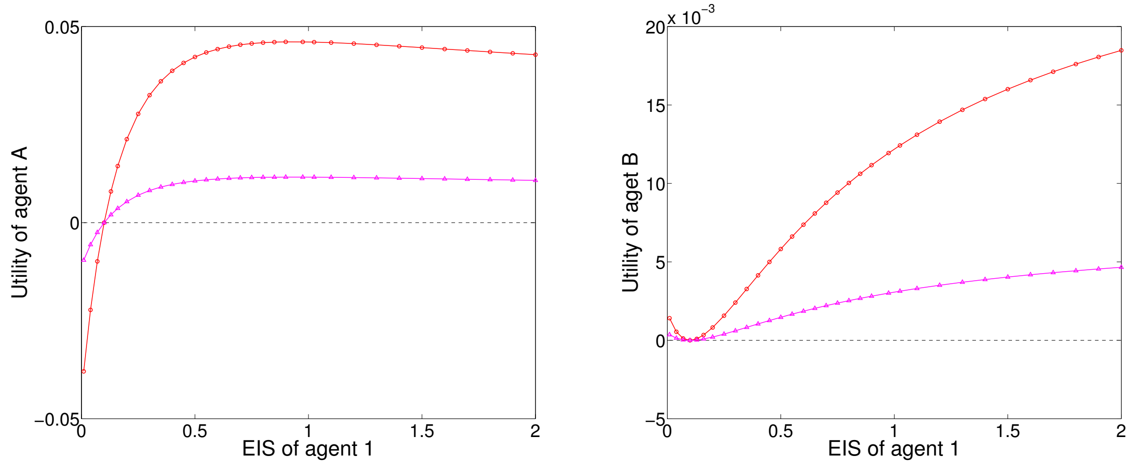


Figure 8: The horizontal axis measures the EIS of agent A. The red line with circle markers and the pink line with triangle markers plot $\frac{U_i\left(c_i^{(L)*};\mathbb{P}_i,\mathcal{F}^{(L)}\right)}{U_i\left(c_i^*;\mathbb{P}_i,\mathcal{F}\right)} - 1$ respectively $\frac{U_i\left(c_i^{(T)*};\mathbb{P}_i,\mathcal{F}^{(T)}\right)}{U_i\left(c_i^*;\mathbb{P}_i,\mathcal{F}\right)} - 1$, that is, the percentage of additional lifetime consumption agent $i \in I$ requires in the economy with late uncertainty resolution in order to be indifferent to the equilibrium consumption allocation he would receive in the economy with an early release of information about the large respectively tiny stock. The vertical axis is measured in percentage points. The left panel illustrates the case of $i = A$ and right panel $i = B$.

and for agent B we assume power utility while agent A has more general recursive preferences. We solve equilibria for all cases of $EIS_A \in (0, 2]$.

As in the previous section we see in figure 8 that agent B always prefers an early release of information but information is only Pareto improving if $EIS_A > \frac{1}{\gamma_A}$. The red line with circle markers and the pink line with triangle markers plot $\frac{U_i\left(c_i^{(L)*};\mathbb{P}_i,\mathcal{F}^{(L)}\right)}{U_i\left(c_i^*;\mathbb{P}_i,\mathcal{F}\right)} - 1$ respectively $\frac{U_i\left(c_i^{(T)*};\mathbb{P}_i,\mathcal{F}^{(T)}\right)}{U_i\left(c_i^*;\mathbb{P}_i,\mathcal{F}\right)} - 1$, that is, the percentage of additional lifetime consumption agent $i \in I$ requires in the economy with late uncertainty reso-

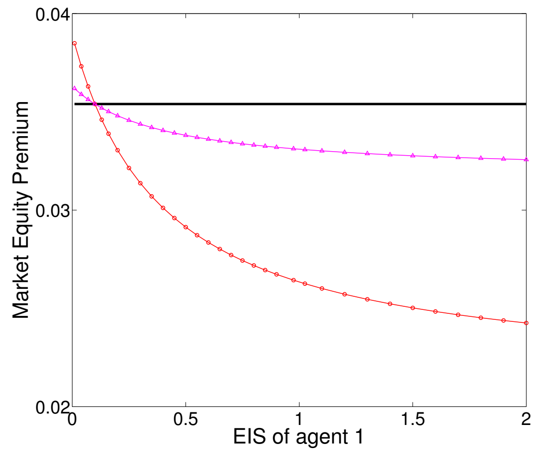


Figure 9: The horizontal axis measures the EIS of agent A. The vertical axis measures the annualized market equity premium of a dividend strip (equation (3)) which pays aggregate endowment at time 1. The black solid line is the equity premium in the economy with late uncertainty resolution, the red line with circle markers and the pink line with triangle markers are the premia in the economies with an early release of information about the large and tiny stock, respectively.

lution in order to be indifferent to the equilibrium consumption allocation he would receive in the economy with an early release of information about the large respectively tiny stock. Not surprisingly, we find that an early release of information about the large stock is more valuable than information about the tiny stock. Intuitively, information about a large stock reveals more information about aggregate endowment and is more valuable for risk-sharing and consumption smoothing compared to a signal about a tiny stock.

Figure 9 illustrates that information about a large stock has a stronger impact on the market equity premium than information about a tiny stock. The black solid

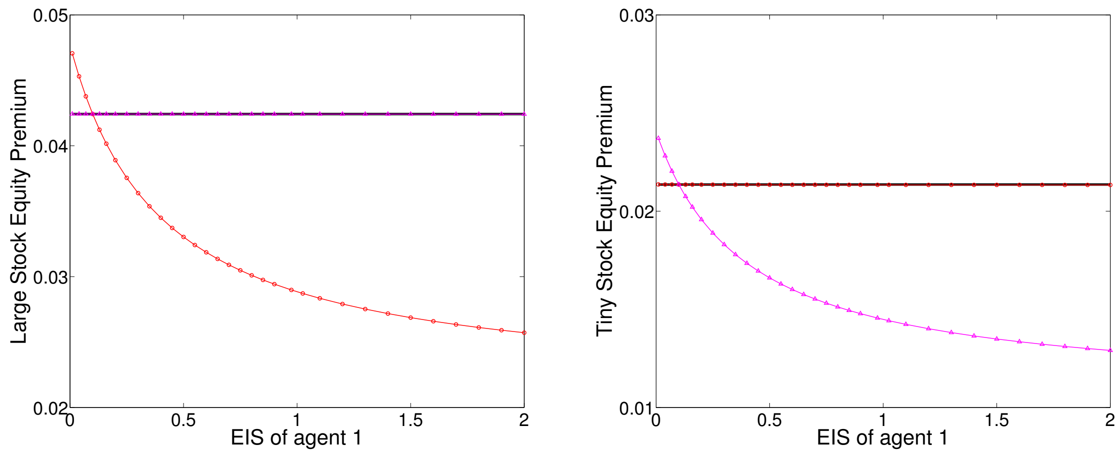


Figure 10: The horizontal axis measures the EIS of agent A. The vertical axis measures the annualized equity premium of a dividend strip (equation (3)). The left and right panels plot the premia of a dividend strip paying $e_{L,1}$ respectively $e_{T,1}$ at time 1. The black solid line is the premium in the economy with late uncertainty resolution, the red line with circle markers and the pink line with triangle markers are the premia in the economies with an early release of information about the large and tiny stock, respectively.

line is the market equity premium in the economy with late uncertainty resolution, the red line with circle markers and the pink line with triangle markers are the premia in the economies with an early release of information about the large and tiny stock, respectively. Notice that the market equity premium is smaller than in figure 7 because large and tiny stocks are independent of each other and aggregate risk as well as the covariations between aggregate endowment and each individual stock are lower. According to figure 8 an early release of information about the large stock has stronger implications on risk-sharing and thus, the pricing kernel and the market equity premium than an early information release about the tiny stock.

Figure 10 reveals that information about the tiny (large) stock has virtually no implications for the equity premium of the large (tiny) stock. Moreover, the large stock pays a higher equity premium than the tiny stock because the large stock introduces more variation in aggregate endowment and the pricing kernel than the tiny stock. Given the two trees are independent of each other the results are not surprising. Figure 10 reinforces the result that information about the large stock matters more for pricing aggregate endowment (market equity premium) than information about the tiny stock.

5 Conclusions

We analyze whether early releases of public information ex-ante affect risk-sharing and pricing in a pure exchange economy. Many public information disclosures such as dividend and other corporate event announcements have no (direct) implications on real investment decisions. Provided real investments are unaffected, it is interesting to know whether investors in secondary markets care about these announcements and what implications information releases have on risk-sharing, welfare and pricing.

If agents have time-additive preferences, homogeneous beliefs and access to complete markets, we prove that agents do not care about information releases and there are no implications for either the equilibrium consumption allocation or pricing.

In the case of heterogeneous beliefs, we show analytically that an early release of information Pareto improves the equilibrium consumption allocation. Agents speculate on their specific beliefs and if risk is resolved early, speculative profits are expected to payoff early in time, which increases expected utility of all agents. Welfare improves in the sense that is under each agent's expectation his respective ex-ante utility increases (see Kim (2012), Brunnermeier et al. (2013) or Gilboa et al. (2013) for other approaches to assess welfare). Our analytical results hold for releases of perfect or imperfect signals and any heterogeneity in beliefs, risk aversions, time preferences and initial endowments. An early information release is particularly beneficial if disagreement is large. Moreover, poorer and less risk averse agents benefit more from early information releases. The welfare implications are the same for information releases about large and small stocks and even idiosyncratic events.

Given time additive preferences, the ex-ante equity premium is not essentially affected by the timing of information releases.

Hirshleifer (1971) showed that an early release of information can have negative implications for risk-sharing and welfare, if agents are not allowed to trade before the state of the world is revealed (markets are effectively incomplete). We show that Hirshleifer's (1971) result is true for the case of homogeneous beliefs but does not always hold if agents' beliefs are heterogeneous. Indeed, if agents have heterogeneous beliefs early information releases can be Pareto improving even if agents are not allowed to trade before the infromation is released.

If agents have recursive preferences, numerical solutions suggest that an early release of information improves risk-sharing. Given a large enough EIS, an early release of information is Pareto improving. Welfare improvements are particularly large if aggregate uncertainty is large or the time period in the model is long. Relatively more risk averse and poorer agents benefit more from early information releases.

In contrast to the case of time additive utilities, pricing is substantially affected if agents have recursive preferences. For large enough levels in the EIS, the ex-ante equity premium is lower if information is released early. This is in contrast to Ross' (1989) finding that the timing of information releases does not matter for ex-ante pricing if the pricing kernel is modelled exogenously. In our paper the pricing kernel is endogenously determined in equilibrium. The timing of information releases crucially affects the equilibrium consumption allocation and the pricing kernel, and therefore pricing. In general equilibrium, Ross' (1989) result only applies to cases of information releases on idiosyncratic events (which is in accordance with his motivating example in the introduction).

Information releases about large stocks have stronger welfare and pricing implications than information releases about tiny stocks.

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Appendix

A Specificaiton of Equilibria

A.1 Karush-Kuhn-Tucker Optimality Conditions

The KKT optimality conditions corresponding to the maximization problem $(P1)$ are,

$$\begin{aligned}
0 &= \frac{\partial U_i\left(c_i^{\diamond}(q);\mathbb{P}_i\right)}{\partial c_{i,0}(k)} - \lambda_i^{\diamond}(q)\, q_{0,k} \quad \forall k \in \mathcal{P}_0 && (23)\\
0 &= \frac{\partial U_i\left(c_i^{\diamond}(q);\mathbb{P}_i\right)}{\partial c_{i,1}(s)} - \lambda_i^{\diamond}(q)\, q_{1,s} \quad \forall s \in \mathbb{S} && (24)\\
0 &\leq \lambda_i^{\diamond}(q), \qquad c_i^{\diamond}(q) \in \mathcal{L}_+ && (25)\\
0 &\geq \sum_{k\in\mathcal{P}_0} q_{0,k}\left(c_{i,0}^{\diamond}(k,q) - e_{i,0}\right) + \sum_{s\in\mathbb{S}} q_{1,s}\left(c_{i,1}^{\diamond}(s,q) - e_{i,s}\right) && (26)\\
0 &= \lambda_i^{\diamond}(q)\left[\sum_{k\in\mathcal{P}_0} q_{0,k}\left(c_{i,0}^{\diamond}(k,q) - e_{i,0}\right) + \sum_{s\in\mathbb{S}} q_{1,s}\left(c_{i,1}^{\diamond}(s,q) - e_{i,s}\right)\right]. && (27)
\end{aligned}$$

A.2 Note on Existence and Global Uniqueness of Equilibrium

First, existence is given because we consider a pure exchange economy with $e_i \in \mathcal{L}_+$ and every consumer has continuous, strictly convex and strongly monotone preferences. Second, local uniqueness is ensured because our economy is regular. Third, homothetic preferences imply that the uncompensated law of demand property is satisfied for every agent and assuming that agents' relative wealths are independent of prices is a sufficient condition for the weak axiom of reveald preferences to hold for the aggregate demand function. The weak axiom is a sufficient condition for the set of normalized equilibrium prices to be convex, and if we have local uniqueness (set of normalized price equilibria is finite), than q^* must be globally unique. Global uniqueness of q^* implies global uniqueness of $c_i^* = c_i^\diamond(q^*)$. Proofs and further discussions can be found in Mas-Colell, Whinston and Green (1995), Propositions 4.C.1, 4.C.2, 17.C.1, 17.D.1 and 17.F.2.

A.3 Time Additive Preferences

Equilibrium in economy with late uncertainty resolution (filtration $\mathcal{F}$)

We use the utility specification (6) in the system of equations (23) through (27) and rearrange to end up with the following system of equations which determine the equilibrium quantities c_A^*, c_B^*, q^* and λ^* in the economy with no early information releases (filtration $\mathcal{F}$),

$$
\begin{aligned}
0 &= \frac{\partial u_A\left(c_{A,0}^*\right)}{\partial c_{A,0}} - \lambda^* \frac{\partial u_B\left(c_{B,0}^*\right)}{\partial c_{B,0}} && (28)\\
0 &= \beta_A \frac{\partial u_A\left(c_{A,1}^*(s)\right)}{\partial c_{A,1}(s)} - \lambda^* \frac{\mathbb{P}_B(s)}{\mathbb{P}_A(s)} \beta_B \frac{\partial u_B\left(c_{B,1}^*(s)\right)}{\partial c_{B,1}(s)} \quad \forall s \in \mathbb{S} && (29)\\
0 &= \sum_{i \in \{A,B\}} c_i^* - \sum_{i \in \{A,B\}} e_i && (30)\\
0 &= c_{A,0}^* - e_{A,0} + \sum_{s \in \mathbb{S}} q_{1,s}^* \left(c_{A,1}^*(s) - e_{A,s}\right) && (31)\\
q_{1,s}^* &= \frac{\beta_A \mathbb{P}_A(s) \frac{\partial u_A\left(c_{A,1}^*(s)\right)}{\partial c_{A,1}(s)}}{\frac{\partial u_A\left(c_{i,0}^*\right)}{\partial c_{A,0}}}, \qquad \forall s \in \mathbb{S} && (32)\\
\lambda^* &> 0, \qquad \left(c_A^*, c_B^*\right) \in \mathcal{L}_+ \times \mathcal{L}_+. && (33)
\end{aligned}
$$

Equilibrium in economy with early uncertainty resolution (filtration $\widehat{\mathcal{F}}$)

As in the previous section we use the utility specification (6) in the system of equations (23) through (27) but make appropriate adjustments to switch from the case with filtration $\mathcal{F}$ to $\widehat{\mathcal{F}}$. $\widehat{c}_A^*$, $\widehat{c}_B^*$, $\widehat{q}^*$ and $\widehat{\lambda}^*$ in the economy with early information releases (filtration $\widehat{\mathcal{F}}$) satisfy,

$$
\begin{aligned}
0 &= \beta_A^t \frac{\partial u_A\left(\widehat{c}_{A,t}^*(s)\right)}{\partial \widehat{c}_{A,t}(s)} - \widehat{\lambda}^* \frac{\mathbb{P}_B(s)}{\mathbb{P}_A(s)} \beta_B^t \frac{\partial u_B\left(\widehat{c}_{B,t}^*(s)\right)}{\partial \widehat{c}_{B,t}(s)}, \quad \forall s \in \mathbb{S}, t \in \{0,1\} \quad (34)\\
0 &= \sum_{i\in\{A,B\}} \widehat{c}_i^* - \sum_{i\in\{A,B\}} e_i \quad (35)\\
0 &= \sum_{s\in\mathbb{S}} q_{0,s}^* \left(\widehat{c}_{A,0}^*(s) - e_{A,0}\right) + \sum_{s\in\mathbb{S}} q_{1,s}^* \left(\widehat{c}_{A,1}^*(s) - e_{A,s}\right) \quad (36)\\
\widehat{q}_{t,s}^* &= \frac{\beta_A^t \mathbb{P}_A(s) \frac{\partial u_A\left(\widehat{c}_{A,t}^*(s)\right)}{\partial \widehat{c}_{A,t}(s)}}{\sum_{s\in\mathbb{S}} \mathbb{P}_A(s) \frac{\partial u_A\left(\widehat{c}_{i,0}^*(s)\right)}{\partial \widehat{c}_{A,0}(s)}}, \qquad \forall s \in \mathbb{S}, t \in \{0,1\} \quad (37)\\
\widehat{\lambda}^* &> 0, \qquad (\widehat{c}_A^*, \widehat{c}_B^*) \in \mathcal{L}_+ \times \mathcal{L}_+. \quad (38)
\end{aligned}
$$

Equilibrium in incomplete markets economy with early uncertainty resolution (filtration $\widehat{\mathcal{F}}$)

Equilibrium conditions for the quantities $\widehat{\widehat{c}}_A{}^*(s)$, $\widehat{\widehat{c}}_B{}^*(s)$, $\widehat{\widehat{R}}^*(s)$ and $\widehat{\widehat{\lambda}}_i{}^*(s)$ are derived directly from the maximization problem (19) and imposing market clearing,

$$
\begin{aligned}
0 &= \beta_A^t \frac{\partial u_A\left(\widehat{\widehat{c}}_{A,t}^*(s)\right)}{\partial \widehat{\widehat{c}}_{A,t}(s)} - \frac{\widehat{\widehat{\lambda}}_A{}^*(s)}{\widehat{\widehat{\lambda}}_B{}^*(s)} \beta_B^t \frac{\partial u_B\left(\widehat{\widehat{c}}_{B,t}^*(s)\right)}{\partial \widehat{\widehat{c}}_{B,t}(s)}, \quad \forall t \in \{0,1\} \quad (39)\\
0 &= \sum_{i\in\{A,B\}} \widehat{\widehat{c}}_{i,t}^*(s) - \sum_{i\in\{A,B\}} e_{i,t}, \quad \forall t \in \{0,1\} \quad (40)\\
0 &= \widehat{\widehat{c}}_{i,0}^*(s) - e_{A,0} + \frac{\widehat{\widehat{c}}_{i,1}^*(s) - e_{A,1}(s)}{\widehat{\widehat{R}}^*(s)} \quad (41)\\
\widehat{\widehat{R}}^*(s) &= \frac{1}{\beta_A} \frac{\frac{\partial u_A\left(\widehat{\widehat{c}}_{A,0}^*(s)\right)}{\partial \widehat{\widehat{c}}_{A,0}(s)_{A,0}}}{\frac{\partial u_A\left(\widehat{\widehat{c}}_{A,1}^*(s)\right)}{\partial \widehat{\widehat{c}}_{A,1}(s)}} \quad (42)\\
\widehat{\widehat{\lambda}}_A{}^*(s) &> 0, \quad \widehat{\widehat{\lambda}}_B{}^*(s) > 0, \qquad \left(\widehat{\widehat{c}}_A^*, \widehat{\widehat{c}}_B^*\right) \in \mathcal{L}_+ \times \mathcal{L}_+. \quad (43)
\end{aligned}
$$

In the special case of log-utility we can derive explicit solutions. The derivation is similar the the proof of Proposition 2. From the budget constraint (41) together with equation (42), $\frac{e_{A,0}}{e_{B,0}} = \frac{e_{A,1}(s)}{e_{B,1}(s)}$ $\forall s \in \mathbb{S}$ and $u_i(x) = \ln(x)$, $\forall i \in I$ we get

$$\frac{\widehat{\widehat{c}}^*_{A,0}(s)}{\widehat{\widehat{c}}^*_{B,0}(s)} = \frac{e_{A,0}(1+\beta_B)}{e_{B,0}(1+\beta_A)}, \forall s \in \mathbb{S},$$

and combining with equation (39) yields

$$\frac{\widehat{\widehat{c}}^*_{A,1}(s)}{\widehat{\widehat{c}}^*_{B,1}(s)} = \frac{e_{A,0}\beta_A(1+\beta_B)}{e_{B,0}\beta_B(1+\beta_A)}, \ \forall s \in \mathbb{S}.$$

Market clearing (40) implies

$$\widehat{\widehat{c}}^*_{i,t}(s) = e_t(s)\frac{e_{i,0}\beta_i^t(1+\beta_j)}{e_{i,0}\beta_i^t(1+\beta_j) + e_{j,0}\beta_j^t(1+\beta_i)},$$

$\forall t \in \{0,1\}, s \in \mathbb{S}, i, j \in I, i \neq j$. Agent $i \in I$'s expected utility at time -1 is

$$\begin{aligned} U_i\left(\widehat{\widehat{c}}^*_i; \mathbb{P}_i\right) &= \ln(e_0) + E^i[\beta_i \ln(e_t(s))] - \ln\left(1 + \frac{e_{j,0}(1+\beta_i)}{e_{i,0}(1+\beta_j)}\right) \\ &\quad - \beta_i \ln\left(1 + \frac{e_{j,0}\beta_j(1+\beta_i)}{e_{i,0}\beta_i(1+\beta_j)}\right), \quad \forall i, j \in I, i \neq j. \end{aligned}$$

A.4 Recursive Preferences

Equilibrium in economy with late uncertainty resolution (filtration $\mathcal{F}$)

We use the utility specification (20) in the system of equations (23) through (27) and rearrange to end up with the following system of equations which determine the equilibrium quantities c^*_A, c^*_B, q^* and λ^* in the economy with no early information releases (filtration $\mathcal{F}$),

$$
\begin{aligned}
0 &= \left(\frac{c_{A,0}^{*}}{U_A\left(c_A^{*};\mathbb{P}_A,\mathcal{F}\right)}\right)^{\rho_A-1} - \lambda^{*}\left(\frac{c_{B,0}^{*}}{U_B\left(c_B^{*};\mathbb{P}_B,\mathcal{F}\right)}\right)^{\rho_B-1} && (44)\\
0 &= \frac{\mathbb{P}_A(s)}{\mathbb{P}_B(s)}\frac{\beta_A}{\beta_B}\frac{\left(U_A\left(c_A^{*};\mathbb{P}_A,\mathcal{F}\right)\right)^{1-\rho_A}\left(c_{A,1}^{*}(s)\right)^{-\gamma_A}}{\left(E^A\left[\left(c_{A,1}^{*}(s)\right)^{1-\gamma_A}\right]\right)^{\frac{1-\rho_A-\gamma_A}{1-\gamma_A}}} && (45)\\
&\quad -\lambda^{*}\frac{\left(U_B\left(c_B^{*};\mathbb{P}_B,\mathcal{F}\right)\right)^{1-\rho_B}\left(c_{B,1}^{*}(s)\right)^{-\gamma_B}}{\left(E^B\left[\left(c_{B,1}^{*}(s)\right)^{1-\gamma_B}\right]\right)^{\frac{1-\rho_B-\gamma_B}{1-\gamma_B}}}, \quad \forall s\in\mathbb{S} \\
0 &= \sum_{i\in\{A,B\}} c_i^{*} - \sum_{i\in\{A,B\}} e_i && (46)\\
0 &= c_{A,0}^{*} - e_{A,0} + \sum_{s\in\mathbb{S}} q_{1,s}^{*}\left(c_{A,1}^{*}(s) - e_{A,s}\right) && (47)\\
q_{1,s}^{*} &= \beta_A\mathbb{P}_A(s)\left(E^A\left[\left(c_{A,1}^{*}(s)\right)^{1-\gamma_A}\right]\right)^{-\frac{1-\rho_A-\gamma_A}{1-\gamma_A}}\frac{\left(c_{A,1}^{*}(s)\right)^{-\gamma_A}}{\left(c_{A,0}^{*}\right)^{\rho_A-1}}, \quad \forall s\in\mathbb{S} && (48)\\
\lambda^{*} &> 0, \qquad \left(c_A^{*},c_B^{*}\right) \in \mathcal{L}_{+}\times\mathcal{L}_{+}. && (49)
\end{aligned}
$$

Equilibrium in economy with early uncertainty resolution (filtration $\widehat{\mathcal{F}}$)

As in the previous section we use the utility specification (21) in the system of equations (23) through (27) and make appropriate adjustments to switch from the case with filtration $\mathcal{F}$ to $\widehat{\mathcal{F}}$. $\widehat{c}_A^{*}$, $\widehat{c}_B^{*}$, $\widehat{q}^{*}$ and $\widehat{\lambda}^{*}$ in the economy with early information releases (filtration $\widehat{\mathcal{F}}$) satisfy,

$$
\begin{aligned}
0 &= \frac{\mathbb{P}_A(s)}{\mathbb{P}_B(s)} \frac{\beta_A^t}{\beta_B^t} \frac{\left(U_A\left(\widehat{c}_A^*; \mathbb{P}_A, \widehat{\mathcal{F}}\right)\right)^{\gamma_A} \left(\widehat{c}_{A,t}^*(s)\right)^{\rho_A - 1}}{\left(\left(\widehat{c}_{A,0}^*(s)\right)^{\rho_A} + \beta_A \left(\widehat{c}_{A,1}^*(s)\right)^{\rho_A}\right)^{-\frac{1-\rho_A-\gamma_A}{\rho_A}}} \\
&\quad -\widehat{\lambda}^* \frac{\left(U_B\left(\widehat{c}_B^*; \mathbb{P}_B, \widehat{\mathcal{F}}\right)\right)^{\gamma_B} \left(\widehat{c}_{B,t}^*(s)\right)^{\rho_B - 1}}{\left(\left(\widehat{c}_{B,0}^*(s)\right)^{\rho_B} + \beta_B \left(\widehat{c}_{B,1}^*(s)\right)^{\rho_B}\right)^{-\frac{1-\rho_B-\gamma_B}{\rho_B}}}, \quad \forall s \in \mathbb{S}, t \in \{0,1\} \quad (50) \\
0 &= \sum_{i \in \{A,B\}} \widehat{c}_i^* - \sum_{i \in \{A,B\}} e_i \quad (51) \\
0 &= \sum_{s \in \mathbb{S}} q_{0,s}^* \left(\widehat{c}_{A,0}^*(s) - e_{A,0}\right) + \sum_{s \in \mathbb{S}} q_{1,s}^* \left(\widehat{c}_{A,1}^*(s) - e_{A,s}\right) \quad (52) \\
\widehat{q}_{t,s}^* &= \frac{\beta_A^t \mathbb{P}_A(s) \frac{\left(\widehat{c}_{A,t}^*(s)\right)^{\rho_A - 1}}{\left(\left(\widehat{c}_{A,0}^*(s)\right)^{\rho_A} + \beta_A \left(\widehat{c}_{A,1}^*(s)\right)^{\rho_A}\right)^{-\frac{1-\rho_A-\gamma_A}{\rho_A}}}}{E^A\left[\frac{\left(\widehat{c}_{A,0}^*(s)\right)^{\rho_A - 1}}{\left(\left(\widehat{c}_{A,0}^*(s)\right)^{\rho_A} + \beta_A \left(\widehat{c}_{A,1}^*(s)\right)^{\rho_A}\right)^{-\frac{1-\rho_A-\gamma_A}{\rho_A}}}\right]}, \quad \forall s \in \mathbb{S}, t \in \{0,1\} \quad (53) \\
\widehat{\lambda}^* &> 0, \qquad (\widehat{c}_A^*, \widehat{c}_B^*) \in \mathcal{L}_+ \times \mathcal{L}_+. \quad (54)
\end{aligned}
$$

Equilibrium in economy with two trees and early uncertainty resolution (filtration $\widehat{\mathcal{F}}$)

The equilibrium $\left\{c_A^{(h)*}, c_B^{(h)*}, q^{(h)*}, \lambda^{(h)*}\right\}$ in the economy with two stocks and early information releases about the large respectively tiny stock can be derived by using the utility function (22) in the system of equations (23) through (27) and making appropriate adjustments for the case with filtration $\widehat{\mathcal{F}}$. $\left\{c_A^{(h)*}, c_B^{(h)*}, q^{(h)*}, \lambda^{(h)*}\right\}$ satisfies,

$$
\begin{aligned}
0 &= \frac{\mathbb{P}_A(k)}{\mathbb{P}_B(k)} \frac{\Phi\left(c_A^{(h)*};\mathbb{P}_A,k\right)\left(c_{A,0}^{(h)*}(k)\right)^{\rho_A-1}}{\left(U_A\left(c_A^{(h)*};\mathbb{P}_A,\mathcal{F}^{(h)}\right)\right)^{-\gamma_A}} \\
&\quad -\lambda^{(h)*}\frac{\Phi\left(c_B^{(h)*};\mathbb{P}_B,k\right)\left(c_{B,0}^{(h)*}(k)\right)^{\rho_B-1}}{\left(U_B\left(c_B^{(h)*};\mathbb{P}_B,\mathcal{F}^{(h)}\right)\right)^{-\gamma_B}}, \qquad \forall k\in\mathcal{P}_0^{(h)} \qquad (55)\\
0 &= \frac{\beta_A}{\beta_B}\frac{\mathbb{P}_A(s|k)}{\mathbb{P}_B(s|k)}\frac{\left(c_{A,0}^{(h)*}(k)\right)^{1-\rho_A}\left(c_{A,1}^{(h)*}(s)\right)^{-\gamma_A}}{\left(E^A\left[\left(c_{A,1}^{(h)*}\right)^{1-\gamma_A}\middle| k\right]\right)^{\frac{1-\gamma_A-\rho_A}{1-\gamma_A}}} \\
&\quad -\frac{\left(c_{B,0}^{(h)*}(k)\right)^{1-\rho_B}\left(c_{B,1}^{(h)*}(s)\right)^{-\gamma_B}}{\left(E^B\left[\left(c_{B,1}^{(h)*}\right)^{1-\gamma_B}\middle| k\right]\right)^{\frac{1-\gamma_B-\rho_B}{1-\gamma_B}}}, \qquad \forall s\in\mathbb{S} \qquad (56)\\
0 &= \sum_{i\in\{A,B\}} c_i^{(h)*} - \sum_{i\in\{A,B\}} e_i \qquad (57)\\
0 &= \sum_{k\in\mathcal{P}_0^{(h)}} q_{0,k}^{(h)*}\left(c_{A,0}^{(h)*}(k)-e_{A,0}\right) + \sum_{s\in\mathbb{S}} q_{1,s}^{(h)*}\left(c_{A,1}^{(h)*}(s)-e_{A,s}\right) \qquad (58)\\
q_{0,k}^{(h)*} &= \frac{\mathbb{P}_A(k)\,\Phi\left(c_A^{(h)*};\mathbb{P}_A,k\right)\left(c_{A,0}^{(h)*}(k)\right)^{\rho_A-1}}{E^A\left[\Phi\left(c_A^{(h)*};\mathbb{P}_A,k\right)\left(c_{A,0}^{(h)*}(k)\right)^{\rho_A-1}\right]}, \qquad \forall k\in\mathcal{P}_0^{(h)} \qquad (59)\\
q_{1,s}^{(h)*} &= q_{0,k}^{(h)*}\beta_A\mathbb{P}_A(s|k)\frac{\left(c_{A,0}^{(h)*}(k)\right)^{1-\rho_A}\left(c_{A,1}^{(h)*}(s)\right)^{-\gamma_A}}{\left(E^A\left[\left(c_{A,1}^{(h)*}\right)^{1-\gamma_A}\middle| k\right]\right)^{\frac{1-\gamma_A-\rho_A}{1-\gamma_A}}}, \ \forall s\in\mathbb{S} \qquad (60)\\
\lambda^{(h)*} &> 0, \qquad \left(c_A^{(h)*},c_B^{(h)*}\right) \in \mathcal{L}_+^{(h)}\times\mathcal{L}_+^{(h)}, \qquad (61)
\end{aligned}
$$

with $\Phi\left(c_i^{(h)};\mathbb{P}_i,k\right) = \left(\left(c_{i,0}^{(h)}(k)\right)^{\rho_i} + \beta_i\left(E^i\left[\left(c_{i,1}^{(h)}\right)^{1-\gamma_i}\middle| k\right]\right)^{\frac{\rho_i}{1-\gamma_i}}\right)^{\frac{1-\gamma_i-\rho_i}{\rho_i}}$, $k\in\mathcal{P}_0^{(h)}$.

B Proofs of Propositions

Proof of Proposition 2

We first derive explicit solutions in the economy with late uncertainty resolution (filtration $\mathcal{F}$). Plugging equation (32) into (31), using the fact that $\frac{e_{A,0}}{e_{B,0}} = \frac{e_{A,1}(s)}{e_{B,1}(s)}$ $\forall s \in \mathbb{S}$ and setting $u_i(x) = \ln(x)$ $\forall i \in I$ we get

$$e_{i,0}\left(1 + \sum_{s\in\mathbb{S}} q^*_{1,s}\right) = c^*_{i,0}\left(1+\beta_i\right),\ \forall i \in I,$$

or

$$\frac{c^*_{A,0}}{c^*_{B,0}} = \frac{e_{A,0}}{e_{B,0}}\frac{1+\beta_B}{1+\beta_A}.$$

Combining with equations (28) and (29) we get

$$\frac{c^*_{A,1}(s)}{c^*_{B,1}(s)} = \frac{e_{A,0}}{e_{B,0}}\frac{\beta_A}{\beta_B}\frac{1+\beta_B}{1+\beta_A}\frac{\mathbb{P}_A(s)}{\mathbb{P}_B(s)},\ \forall s \in \mathbb{S}.$$

Market clearing condition (30) implies

$$\begin{aligned} c^*_{i,0} &= e_0\frac{e_{i,0}\left(1+\beta_j\right)}{e_{i,0}\left(1+\beta_j\right)+e_{j,0}\left(1+\beta_i\right)} \\ c^*_{i,1}(s) &= e_1(s)\,\frac{e_{i,0}\beta_i\left(1+\beta_j\right)\mathbb{P}_i(s)}{e_{i,0}\beta_i\left(1+\beta_j\right)\mathbb{P}_i(s)+e_{j,0}\beta_j\left(1+\beta_i\right)\mathbb{P}_j(s)}, \end{aligned}$$

$\forall s \in \mathbb{S}, i, j \in I \times I, i \neq j$. Agent $i \in I$'s expected utility at time -1 is

$$\begin{aligned} U_i\left(c^*_i;\mathbb{P}_i\right) =& \ln\left(e_0\right) + \beta_i E^i\left[\ln\left(e_1(s)\right)\right] - \ln\left(1+\frac{e_{j,0}\left(1+\beta_i\right)}{e_{i,0}\left(1+\beta_j\right)}\right) \\ &- \beta_i E^i\left[\ln\left(1+\frac{e_{j,0}\beta_j\left(1+\beta_i\right)\mathbb{P}_j(s)}{e_{i,0}\beta_i\left(1+\beta_j\right)\mathbb{P}_i(s)}\right)\right],\ \forall i,j \in I, i \neq j. \end{aligned}$$

We now turn to the economy with early uncertainty resolution, where a perfect signal about the future state of the economy is observed at time 0 (filtration $\widehat{\mathcal{F}}$). As before, plugging equation (37) into (36), using the fact that $\frac{e_{A,0}}{e_{B,0}} = \frac{e_{A,1}(s)}{e_{B,1}(s)}$ $\forall s \in \mathbb{S}$ and setting $u_i(x) = \ln(x)$ $\forall i \in I$ we get

$$\sum_{s\in\mathbb{S}} \mathbb{P}_i(s) \frac{1}{\widehat{c}^*_{i,0}(s)} = \frac{1+\beta_i}{e_{i,0}\left(1+\sum_{s\in\mathbb{S}} \widehat{q}^*_{1,s}\right)}, \ \forall i \in I,$$

and combining with equation (34) for $t = 0$ and summing over $s \in \mathbb{S}$ we have

$$\widehat{\lambda}^* = \frac{\sum_{s\in\mathbb{S}} \mathbb{P}_A(s) \frac{1}{\widehat{c}^*_{A,0}(s)}}{\sum_{s\in\mathbb{S}} \mathbb{P}_B(s) \frac{1}{\widehat{c}^*_{B,0}(s)}} = \frac{e_{B,0}(1+\beta_A)}{e_{A,0}(1+\beta_B)}.$$

Plugging back into equation (34) and imposing market clearing (35) yields

$$\widehat{c}^*_{i,t}(s) = e_t(s) \frac{e_{i,0}\beta_i^t (1+\beta_j) \mathbb{P}_i(s)}{e_{i,0}\beta_i^t (1+\beta_j) \mathbb{P}_i(s) + e_{j,0}\beta_j^t (1+\beta_i) \mathbb{P}_j(s)},$$

$\forall t \in \{0,1\}, s \in \mathbb{S}, i,j \in I, i \neq j$. Agent $i \in I$'s expected utility at time -1 is

$$\begin{aligned} U_i(\widehat{c}^*_i; \mathbb{P}_i) = \ln(e_0) + E^i[\beta_i \ln(e_t(s))] - E^i\left[\ln\left(1 + \frac{e_{j,0}(1+\beta_i)\mathbb{P}_j(s)}{e_{i,0}(1+\beta_j)\mathbb{P}_i(s)}\right)\right] \\ - \beta_i E^i\left[\ln\left(1 + \frac{e_{j,0}\beta_j(1+\beta_i)\mathbb{P}_j(s)}{e_{i,0}\beta_i(1+\beta_j)\mathbb{P}_i(s)}\right)\right], \ \forall i,j \in I, i \neq j. \end{aligned}$$

The result $U_i(\widehat{c}^*_i; \mathbb{P}_i) \geq U_i(c^*_i; \mathbb{P}_i), \forall i \in I$ is due to Jensen's inequality (see main text just below Proposition 2).

Proof of Proposition 3

In the following we suppress the superscript star to indicate equilibrium quantities. The technical gist behind the results of this Proposition is Jensen's inequality. First, we establish an important convexity of the utility function in equilibrium. To this end it suffices to examine the simpler system of FOC and resource constraint in period $t = 0$ for the economy without early information releases (below, u' denotes marginal utility),

$$u'_A(c_{A,0}) = \lambda u'(c_{B,0}), \qquad c_{A,0} + c_{B,0} = e_0,$$

and compute the second derivative of the utility *at equilibrium* with respect to Pareto weight,[18]

$$\begin{aligned}\frac{d^2 u_A\left(c_{A,0}\right)}{d\lambda^2} &= \frac{d}{d\lambda}\left(u_A'\left(c_{A,0}\right)\frac{dc_{A,0}}{d\lambda}\right) \\ &= u_A''\left(c_{A,0}\right)\left(\frac{dc_{A,0}}{d\lambda}\right)^2 + u_A'\left(c_{A,0}\right)\frac{d^2 c_{A,0}}{d\lambda^2},\end{aligned} \tag{62}$$

where u'' denotes second-order derivative with respect to consumption. A derivation identical to that of (16) yields

$$\frac{dc_{A,0}}{d\lambda} = \frac{1}{\lambda\left(\frac{u_A''\left(c_{A,0}\right)}{u_A'\left(c_{A,0}\right)} + \frac{u_B''\left(c_{B,0}\right)}{u_B'\left(c_{B,0}\right)}\right)} = -\lambda^{-1}\left(\frac{\gamma_A}{c_{A,0}} + \frac{\gamma_B}{c_{B,0}}\right)^{-1}, \tag{63}$$

where in the last equality we made use of the explicit form of CRRA utilities. Taking the derivative with respect to λ of the above expression one more time we obtain

$$\frac{d^2 c_{A,0}}{d\lambda^2} = \lambda^{-2}\left(\frac{\gamma_A}{c_{A,0}} + \frac{\gamma_B}{c_{B,0}}\right)^{-3}\left[\frac{\gamma_A}{c_{A,0}}\left(\frac{\gamma_A}{c_{A,0}} + \frac{2\gamma_B}{c_{B,0}} + \frac{1}{c_{A,0}}\right) + \frac{\gamma_B(\gamma_B - 1)}{c_{B,0}^2}\right]. \tag{64}$$

Clearly, when $\gamma_B > 1$, agent A's equilibrium consumption is both decreasing and convex in λ. Substituting (63), (64) into (62) yields

$$\frac{d^2 u_A\left(c_{A,0}\right)}{d\lambda^2} = \lambda^{-2}\left(\frac{\gamma_A}{c_{A,0}} + \frac{\gamma_B}{c_{B,0}}\right)^{-3} c_{A,0}^{-\gamma_A}\left[\frac{\gamma_A}{c_{A,0}}\left(\frac{\gamma_B}{c_{B,0}} + \frac{1}{c_{A,0}}\right) + \frac{\gamma_B(\gamma_B - 1)}{c_{B,0}^2}\right].$$

Again, when $\gamma_B > 1$, agent A's utility in period $t = 0$ is both decreasing and strictly convex in λ. Note that our derivation extends immediately to the more elaborate system of FOC and resource constraint in the setting of early information releases and for any state and each point in time. Indeed, the FOC in state $s \in \mathbb{S}$ at time

[18] Note that equilibrium consumptions are solutions of the system of FOC and market clearing conditions, so they are functions of both Pareto weight and aggregate endowment.

$t = 0$ of the early uncertainty resolution setting can be rewritten as

$$\begin{aligned}
&\mathbb{P}_A(s)u'\left(\widehat{c}_{A,0}\left(s\right)\right) = \widehat{\lambda}\mathbb{P}_B(s)u'\left(\widehat{c}_{B,0}\left(s\right)\right) \\
\iff\ & u'(\widehat{c}_{A,0}(s)) = \widehat{\lambda}\frac{\mathbb{P}_B(s)}{\mathbb{P}_A(s)}u'(\widehat{c}_{B,0}(s)) \equiv \widehat{\lambda}(s)u'(\widehat{c}_{B,0}(s)).
\end{aligned}$$

Now, for each state $s \in \mathbb{S}$, agent A's utility being decreasing and convex in $\widehat{\lambda}(s) \equiv \widehat{\lambda}\frac{\mathbb{P}_B(s)}{\mathbb{P}_A(s)}$ is equivalent to A's utility being decreasing and and convex in $\widehat{\lambda}$ because the Radon-Nikodym derivative $\frac{\mathbb{P}_B(s)}{\mathbb{P}_A(s)}$ is just a constant parameter for each $s \in \mathbb{S}$.

Next, we apply Jensen's inequality using the fact that agent A's utility function is strictly convex in the implicit variable $\widehat{\lambda}\frac{\mathbb{P}_B(s)}{\mathbb{P}_A(s)}$,

$$\begin{aligned}
\widehat{U}_{A,0} &= E^A\left[u(\widehat{c}_{A,0}(s))\right] = E^A\left[u_A\left(\widehat{\lambda}\frac{\mathbb{P}_B(s)}{\mathbb{P}_A(s)}\right)\right] \\
&> u_A\left(E^A\left[\widehat{\lambda}\frac{\mathbb{P}_B(s)}{\mathbb{P}_A(s)}\right]\right) = u_A\left(c_{A,0}(\widehat{\lambda})\right),
\end{aligned} \tag{65}$$

where $c_{A,0}(\widehat{\lambda})$ in the last equality[19] denotes agent A's equilibrium consumption in the *late uncertainty resolution* setting valued at some *exogenously given* Pareto weight $\widehat{\lambda}$.

Recall that agent A's utility decreases with Pareto weight in the late uncertainty resolution setting. Consequently, if $\lambda > \widehat{\lambda}$, we have $u_A\left(c_{A,0}(\widehat{\lambda})\right) > u_A\left(c_{A,0}(\lambda)\right) = u_A(c_{A,0})$, where the last equality arises because λ is the equilibrium (endogenous) Pareto weight of the economy without early information releases. Combining this with (65) immediately yields the first result of Proposition 3 that, $\widehat{U}_{A,0} > u_A(c_{A,0}) = U_{A,0}$.

To prove the second result of the Proposition, it suffices to use a symmetric argument to see that agent B's utility is decreasing and strictly convex in $\frac{1}{\lambda}$ and

[19]This last equality arises because $E^A\left[\widehat{\lambda}\frac{\mathbb{P}_B(s)}{\mathbb{P}_A(s)}\right] = \sum_{s\in\mathbb{S}}\mathbb{P}_A(s)\widehat{\lambda}\frac{\mathbb{P}_B(s)}{\mathbb{P}_A(s)} = \widehat{\lambda}\sum_{s\in\mathbb{S}}\mathbb{P}_B(s) = \widehat{\lambda}$.

$\frac{1}{\lambda}\frac{\mathbb{P}_A(s)}{\mathbb{P}_B(s)}$. Then, a derivation identical to the above analysis shows that $\widehat{U}_{B,0} > U_{B,0}$ when $\widehat{\lambda} > \lambda$.

Finally, in the case of $\widehat{\lambda} = \lambda$, both first and second results of the Proposition apply and jointly imply simultaneously $\widehat{U}_{A,0} > U_{A,0}$ and $\widehat{U}_{B,0} > U_{B,0}$ ■

Proof of Proposition 4

To demonstrate the relevance of information in a heterogeneous-belief setting we introduce an auxiliary (fictitious) economy as follows.

Auxiliary economy: After having traded to settle with the optimal consumption plans under a late uncertainty resolution premise agents are made aware that a perfect signal is about to arrive. Agents would trade again to cope with and exploit the new (but still ex-ante) circumstance of perfect foresight. However, the initial endowment claims $\{e_{A,0}, e_{A,1}(s)\}$,$\{e_{B,0}, e_{B,1}(s)\}$ have already changed and become $\{c_{A,0}, c_{A,1}(s)\}$,$\{c_{B,0}, c_{B,1}(s)\}$, and the latter are agents' new endowments before re-trading. Hence, effectively, this auxiliary economy is a setting with early informaiton releases but with initial endowments equal to the equilibrium consumptions $\{c_{A,0}, c_{A,1}(s)\}$,$\{c_{B,0}, c_{B,1}(s)\}$ of the original setting with no early information releases. We use a "doubled hat" to denote quantities in the equilibrium of the auxiliary economy, whose FOC and market clearing condition at $t = 0$ and $t = 1$ are respectively,

$$\begin{cases} \mathbb{P}_A(s)u'_A\left(\widehat{\widehat{c}}_{A,0}(s)\right) = \widehat{\widehat{\lambda}}\mathbb{P}_B(s)u'_B\left(\widehat{\widehat{c}}_{B,0}(s)\right) \\ \widehat{\widehat{c}}_{A,0}(s) + \widehat{\widehat{c}}_{B,0}(s) = c_0 \end{cases}$$
$$\begin{cases} \beta_A\mathbb{P}_A(s)u'_A\left(\widehat{\widehat{c}}_{A,1}(s)\right) = \widehat{\widehat{\lambda}}\beta_B\mathbb{P}_B(s)u'_B\left(\widehat{\widehat{c}}_{B,1}(s)\right) \\ \widehat{\widehat{c}}_{A,1}(s) + \widehat{\widehat{c}}_{B,1}(s) = c_1(s) \end{cases}, \tag{66}$$

Because equilibrium prices must preserve the budgets of each agent, the no-trade configuration $\{c_{A,0}, c_{A,1}(s)\}$,$\{c_{B,0}, c_{B,1}(s)\}$ is budget-feasible at the auxiliary equilibrium prices $\{\widehat{\widehat{q}}_0(s), \widehat{\widehat{q}}_1(s)\}$. As agents are price takers in a competitive equilibrium, the auxiliary equilibrium allocation is optimal for each agent (among all budget-feasible configurations), including the initial allocation $\{c_{A,0}, c_{A,1}(s)\}$,$\{c_{B,0}, c_{B,1}(s)\}$. The mere fact that agents can choose to trade to reach the auxiliary equilibrium allocation from the initial early uncertainty resolution equilibrium configuration $\{c_{A,0}, c_{A,1}(s)\}$,$\{c_{B,0}, c_{B,1}(s)\}$, equilibrium then implies that the auxiliary equilibrium is welfare-improving for *both* agents compared to the original early uncertainty resolution equilibrium allocation,

$$\widehat{\widehat{U}}_A \geq U_A; \qquad \widehat{\widehat{U}}_B \geq U_B, \tag{67}$$

with $\widehat{\widehat{U}}_i \equiv U_i\left(\widehat{\widehat{c}}_i^*; \mathbb{P}_i\right), U_i \equiv U_i\left(c_i^*; \mathbb{P}_i\right), \forall i \in \{A, B\}$.

Suppose $\widehat{\widehat{\lambda}} = \widehat{\lambda}$: In this case the equilibrium of the auxiliary economy coincides with that of the economy with early information releases because both belong to the early uncertainty resolution setting and thus, have identical systems of FOC and market clearings. Consequently, (67) coincides with and thus, proves (18).

Suppose $\widehat{\widehat{\lambda}} \neq \widehat{\lambda}$: Hereafter we just need to consider this case of unequal Pareto weights. There are four distinct and exhaustive scenarios concerning comparative statics of agents' welfare in the two economies with and without early information releases listed below,

$$(a)\begin{cases} U_A > \widehat{U}_A \\ U_B > \widehat{U}_B \end{cases}, \quad (b)\begin{cases} U_A > \widehat{U}_A \\ U_B < \widehat{U}_B \end{cases}, \quad (c)\begin{cases} U_A < \widehat{U}_A \\ U_B > \widehat{U}_B \end{cases}, \quad (d)\begin{cases} U_A < \widehat{U}_A \\ U_B < \widehat{U}_B \end{cases}, \tag{68}$$

with $\widehat{U}_i \equiv U_i\left(\widehat{c}_i^*; \mathbb{P}_i\right), \forall i \in \{A, B\}$. We will in turn rule out scenarios (a), (b), (c), leaving (d) as the only viable scenario, and thus, prove the Proposition.

Ruling out scenario (a) in (68): We first note that as a consequence of the relation between equilibrium consumptions and Pareto weights established in the text, the current premise of $\widehat{\widehat{\lambda}} \neq \hat{\lambda}$ implies that one and only one set of the following inequalities must hold,

$$(i)\ \widehat{\widehat{\lambda}} < \widehat{\lambda} \implies \begin{cases} \widehat{\widehat{U}}_A > \widehat{U}_A \\ \widehat{\widehat{U}}_B < \widehat{U}_B \end{cases}, \qquad (ii)\ \widehat{\widehat{\lambda}} > \widehat{\lambda} \implies \begin{cases} \widehat{\widehat{U}}_A < \widehat{U}_A \\ \widehat{\widehat{U}}_B > \widehat{U}_B \end{cases}. \tag{69}$$

But a combination of (67) and scenario *(a)* in (68) shows that $\widehat{\widehat{U}}_A > \widehat{U}_A$ *and* $\widehat{\widehat{U}}_B > \widehat{U}_B$, which violates *both* sets *(i) and (ii)* in (69). As a result, we rule out scenario *(a)* in (68) by contradiction.

Ruling out scenario (b) in (68): Assume that scenario *(b)* holds true, which implies $\frac{U_A}{\widehat{U}_A}\frac{\widehat{U}_B}{U_B} > 1$. Then it follows from equation (15) that

$$\frac{\lambda}{\widehat{\lambda}} = \frac{U_A}{\widehat{U}_A}\frac{\widehat{U}_B}{U_B} > 1 \implies \lambda > \widehat{\lambda}. \tag{70}$$

Consequently, the result *1.* of Proposition 3 implies a relation between agent A's expected utilities in period $t = 0$ of the two economies with early and late uncertainty resolution,

$$\widehat{U}_{A0} > U_{A0}.$$

On the other hand, making use of relation (17), $\lambda > \widehat{\lambda}$ in (70) also implies that in period $t = 1$,

$$\widehat{U}_{A1} > U_{A1}.$$

Combining the last two equalities, we see that agent A's expected utility (over both periods) is higher in the economy with early information releases than it is in the economy with late uncertainty resolution; $\widehat{U}_A > U_A$. But clearly, this contradicts the currently assumed scenario *(b)* in (68). Accordingly, scenario *(b)* is ruled out by contradiction.

Ruling out scenario (c) in (68): The arguments here are symmetric to those employed to rule out scenario *(b)* above. First, scenario *(c)* and equation (15) imply $\widehat{\lambda} > \lambda$. Second, it follows from this and result 2. of Proposition 3 that $\widehat{U}_{B,0} > U_{B,0}$. Third, $\widehat{\lambda} > \lambda$ together with relation (17) also implies $\widehat{U}_{B,1} > U_{B,1}$. Fourth, the last two arguments yield an inequality for agent B's utility over both periods; $\widehat{U}_B > U_B$. Finally, this is in conflict with the thesis maintained in scenario *(c)* in (68). By contradiction, scenario *(c)* is ruled out.

Thus, by elimination, the only remaining scenario *(d)* must be true. That is, $\widehat{U}_A > U_A$ *and* $\widehat{U}_B > U_B$, which proves the Proposition 4 ■