Nontraded Sector Growth Risks and Economic Sizes in International Asset Pricing *

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Abstract

Output fluctuations in nontraded sectors are a primary country-specific risk factor because nontraded outputs are consumed domestically. While nontraded sector growth risks are mostly non-diversifiable, they can be partially mitigated by international trades in other sectors. The mitigation decreases with the host country's size because a larger economy needs to execute a proportionally larger trade to substitute for losses in its nontraded consumption. In interest rate markets, fluctuations in the growth of industries with higher nontradability feed greater risk to the economy and lower interest rates. In currency markets, these fluctuations generate large currency premia and explain why known funding and investment currencies are associated with economies of both small and large sizes.

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1 Introduction

The rational theory and practice of asset pricing center around fundamental principles of the risk return tradeoff, diversification, and no arbitrage. Movements in an economy's nontraded-sector output should play a key role in the determination of domestic asset prices and their differentials across economies, because these risks are not easily diversified even in an arbitrage-free and complete international market. This paper shows that the nontraded output growth risk is indeed an important determinant of international asset prices. We adopt a consumption-based equilibrium economic setting, with multiple countries, nontraded and traded goods, and trade costs. Our setting features and accounts for *partially traded* goods and services as they actually are in reality. This property allows us to estimate the effects of nontraded output risk that are robust to possible classification errors in macro data. Using data from the Organisation for Economic Co-operation and Development (OECD) economies, we verify new implications of nontraded output growth risk for the interest rate and currency markets.

The main insight of this paper is that the nontradability of a country-specific output amplifies the impact of the output's risk on the host country of any size. From this insight follow all our key conceptual results, which are empirically verified in the paper. First, at the country level, the fluctuations in gross domestic product (GDP) growth of less open-to-trade economies pose greater risk, induce higher precautionary savings motives, and lower home interest rates relative to a cross section of economies. Second, at the industry level, the fluctuations in the output growth of less traded industries also place stronger downward pressure on interest rates. Third, the nontraded output growth risk regulates consumption allocation, moves investors' marginal utilities and exchange rates simultaneously, breaks the uncovered interest rate parity, and generates currency premia. In currency markets, the carry trade strategies that expose investors to larger nontraded output growth risk offer higher returns on average. Fourth, the nontraded output risk matters for the host country's asset pricing both on its own and on its coupling with the country's size. In contrast, country-specific *traded* output growth risks are less prominent, because they are subject to diversification via international trades.

The nontraded sector produces goods and services that cannot be consumed outside of the host country. It includes some important sectors to the economy, such as wholesale and retail trade, hotels and restaurants, real estate, financial intermediaries, transportation, information and communications, art and entertainment, and domestic business activities. Three key features of nontraded outputs stand out. First, nontraded outputs feed the lion's share to the GDP and national aggregate consumption in all countries. Figure 1 shows that the ratio of real nontraded output over GDP is substantial across the OECD economies, ranging from 0.45 to 0.8, and almost 0.7 for the U.S. Second, the *tradabilities*, commonly measured as (one half of) the ratio of total import plus export over output, of key nontraded industries are indeed low. Table 1 shows that the tradabilities of various different nontraded sectors are mostly in the order of few percentage points, and rarely exceed 15% across a host of countries. Third, the magnitude of nontraded output growth risk is largely unrelated to the size of host countries. Figure 2 shows a heterogeneous pattern between the relative size (measured as a country's real GDP over OECD's total real GDP) and nontraded output growth volatility across countries, with the U.S. having a large size and a small nontraded output volatility in one extreme, and Switzerland having a small size and a large nontraded output volatility in the other. These stylized facts imply that the nontraded output growth volatilities would pose a major source of risk to economies of all sizes, and should be reflected in the interest rates, exchange rates and currency returns. We discuss these three asset pricing quantities in turn.

First, the primary force underlying the impact of nontraded growth risk on interest rates is the precautionary savings motives of market participants. Under these motives, volatilities of the home nontraded output, trade partners' nontraded outputs, and the global (aggregate) traded output, all act to depress the home interest rate because these volatilities induce uncertainties in the home consumptions. However, as mentioned above, while the country-specific traded output risk is largely internationalized and diversified in the global pool of traded goods, the nontraded output risk is primarily internalized. Consequently, the home nontraded output volatility impacts the home interest rate directly and more strongly than does the home-specific traded output volatility. To the extent that a country is able to mitigate the nontraded output risk by substituting and adjusting the traded consumption, this adjustment is hindered by the country's size (larger countries face larger and more challenging traded consumption adjustments). Hence, the risk mitigation is partial, and presents a second and indirect impact of the nontraded output volatility coupled with the country's size on the interest rate.

Second, the primary force underlying the impact of nontraded growth risk on the exchange rate is the transmission of shocks across countries. Beyond their dominant impacts at home, nontraded output fluctuations are an important source of risk because they also matter for *all* trade partners of the home country. The transmission of home nontraded output shocks starts with these shocks being substituted and offset imperfectly by adjustments in the home traded consumption. These adjustments are facilitated by international trades, and induce fluctuations in the traded consumptions of home's trade partners by market clearing in traded goods. As the exchange rate characterizes the ratio of two countries' price numeraires, i.e., the difference of two country-specific pricing kernels (SDFs), nontraded output shocks induce fluctuations in the exchange rate. Furthermore, a country's nontraded output risk impacts exchange rate both on its own (via its direct impact on the home SDF) and on its coupling with the home size (via its indirect impact on international trade and foreign SDF).

Third, the nontraded output risks is an important factor behind carry trade profits because it induces simultaneous movements in (hence, the correlation between) SDFs and the exchange rate. Why do certain currency pairs tend to generate profits, whereas others incur losses in the currency market? Let us consider a strategy of borrowing the home currency and lending the foreign currency. An adverse shock to the foreign nontraded output simultaneously causes the foreign currency to appreciate (i.e., a surge in the foreign consumption basket value due to the scarcity of foreign nontraded goods) and the home traded consumption to drop (i.e., a surge in the foreign traded consumption to substitute for the foreign nontraded output loss). That is, with respect to the foreign nontraded risk, this strategy pays well when home investors highly value the consumption, and vice versa. From the perspective of home investors, this carry trade is a hedge against foreign nontraded output shocks, and thus commands a low, possibly negative, expected excess return to home investors with respect to this risk. Note that the adverse foreign nontraded shock induces the foreign currency's appreciation on its own, and the home traded consumption's drop via its coupling with the foreign country's size. Therefore, this carry trade expected return is lower when the foreign nontraded sector is riskier or its economy is larger. By the same argument, this carry trade is not a hedge against the home nontraded output growth risk, and thus commands a high expected return with respect to this risk. The overall expected profit (or loss) of the carry trade is determined by whether the home (or foreign) nontraded output growth risk, mediated by their respective sizes, dominates. Specifically, when home nontraded output sector is sufficiently more volatile and larger than that of the foreign counterpart, short the home and long the foreign currency tend to generate positive expected returns to compensate home investors for bearing the dominating home nontraded risk in the carry trades, and vice versa.

Conceptually, it is important to differentiate the roles of nontraded output growth risks and economy's size in international asset pricing. The nontraded output growth risk enters the setting both as a coupled factor with economy's size and as a stand-alone factor. A country's nontraded output growth risk couples with its size when this risk propagates and affects other countries via consumption substitution, adjustments, and international trades. In this process, the home nontraded output growth risk is *amplified* by the home size in its impact on foreign traded consumptions and foreign SDFs, but is *subdued* by the home size in its impact on the home traded consumption and home SDF. Apart from this coupling, a country's nontraded output growth volatility on its own is a stand-alone risk factor to the host country because the nontraded output is consumed entirely at home. The overall impact of the nontraded output growth risk combines contributions from these two factors.

Empirically, nontraded output risks do not align with sizes across economies. We observe from Figure 2 that among OECD countries, the U.S. (US), Japan (JP) are among largest economies but have lowest nontraded output volatilities, whereas Switzerland (CH) is in the opposite. Because the associated currencies (U.S. Dollar USD, Japanese Yen JPY and Swiss Franc CHF) are known "funding" currencies in FX markets, this observation indicates that economic sizes alone do not explain the funding characteristic of currencies. In this regard, Figure 3 exhibits a remarkable pattern that funding currencies tend to occupy the large-value end of the spectrum of the *product* of nontraded output risk and size. Whereas, "investment" currencies (among them, Australian Dollar AUD, and New Zealand Dollar NZD) tend to occupy the small-value end of the spectrum. This pattern is consistent with our carry trade rationale above that long positions in these funding currencies are hedges (against funding currencies' large coupling of nontraded output risks and sizes) and deliver low expected returns.[1] Figure 6 confirms that the coupling of size and nontraded output growth risk, but not size or nontraded output growth risk alone, is a key determinant of carry trade returns.

We now discuss practical aspects of modeling the tradability of goods and describe empirical designs and results of the paper. First, as no goods are either perfectly traded or perfectly nontraded, their output data are necessarily subject to classification and measurement errors. Second, frictions in trade may hamper the diversification in the traded commodity markets. Third, the relationship between output nontradability and asset prices are of equilibrium nature. These challenges determine our approach to investigate the relationship between nontraded output risks and international asset prices. On the modeling side, we adopt an intuitive "iceberg transport cost"

[1] That is, on the flip side, profitable carry trade strategies take short positions in (i.e., borrow) funding currencies, are exposed to large nontraded output risks, and deliver high expected returns.

to address the partial tradability and equilibrium nature of outputs by different economies. In the literature, the iceberg transport cost refers to a trade friction due to which the amount of goods loss in the transition is proportional to amount of goods initially shipped. A higher transportation cost for an output effectively means that this output is less tradable. Therefore, a variation in the transport cost induces simultaneous changes in the tradability, trade volume and price of outputs in equilibrium. This parsimonious modeling approach aims to provide intuitions about the equilibrium relationship between the above quantities. Guided by this intuitive approach, our empirical analysis constructs the model's testable implications on the equilibrium relationship between observable output tradabilities, their risks and asset returns, but does not calibrate a model of transportation cost per se. The empirical tests employ and address the heterogeneity in the (partial) tradabilities of industries up to 4-digit classification codes (ISIC 4).[2] At this fine resolution level of outputs and their tradabilities, the test approach described below is more tractable than a calibration approach.

We employ several variants of the relative magnitude between trade volumes (import and export) and outputs of a good to measure its tradability and effects in tests. The first test concerns interest rates and output growth risk at the industry level. We regress real interest rates on output growth volatilities of various industries, their tradabilities, and the interaction term, while controlling for other variables. Table 5 shows that across OECD economies and on average, one standard deviation in output growth depresses the real interest rate by an additional 5% when the output's classification changes from perfectly traded to perfectly nontraded. Another test shows a similar result; the volatility of GDP has greater effect on home interest rates when the home economy is less open to trades (i.e., having lower ratio of national trade over GDP).

The next tests concern currency trade strategies, which sort currencies according to some characteristics to verify whether these characteristics are priced in currency markets. We first perform an one-dimensional sort of OECD currencies into four portfolios based on one of the following three country-specific characteristics, namely, (i) variance of nontraded output growth, (ii) size (or GDP/total OECD GDP), and (iii) their product. We then form four carry trade strategies, each borrows USD and lends equally in currencies of the corresponding portfolio. Figure 6 exhibits monotonic carry trade mean returns for portfolios sorted on the nontraded output risk coupled with size, indicating the pricing significance of this coupling factor in currency markets. The long-short strategy on the top and bottom portfolios delivers a mean return of 3.23% and a Sharpe ratio of approximately 45%. For robustness, we then perform a two-dimensional sort of OECD currencies

[2]ISIC 4 stands for the International Standard Industrial Classification of All Economic Activities Revision 4 codes.

into 2-by-2 portfolios based on both variance of nontraded output growth and size. Figure 7 shows that, controlling for (small or large) size, lending currencies of economies of higher nontraded output risk offers lower mean returns. This pattern is consistent with the above rationale that these carry trades are hedges to the nontraded output risk of countries, whose currencies are being long in the strategy.

Our analysis naturally suggests a two-factor pricing model for each country. The factors are nontraded and traded *consumption* growths. The nontraded output, by its nature, is essentially the nontraded consumption and largely internalized within the country. Consequently, shocks to the nontraded output are a fundamental risk which necessarily commands a positive factor price. In contrast, the country's traded consumption is an equilibrium result of several offsetting effects on the fundamentals (traded and nontraded endowments) of all countries in the model. As the result, the sign of the factor price associated with the movements in the country's traded consumption is an empirical matter. Using carry trade portfolios as test assets in a two-stage generalized method of moment (GMM) procedure, Table 9 reports a statistically significant and positive estimate of 30 basis points for the nontraded consumption factor price for the U.S. economy. This estimate quantifies the compensation for bearing U.S. nontraded output risk to investors.

Related Literature: The current paper contributes to a vibrant strand of international finance literature that attempts to rationalize currency premia as risk-adjusted excess returns. Various structural features and risk factors have been identified, among others, fundamental consumption risks in Lustig and Verdelhan (2007), recursive preferences in Colacito and Croce (2011) and Colacito et al. (2018), habit formations in Verdelhan (2010) and Stathopoulos (2017), incomplete markets and risk sharing in Sarkissian (2003), global imbalance risks in Della Corte et al. (2016), commodity versus final good productions in Ready et al. (2017), geopolitics and trade gravities in Lustig and Richmond (2019), and trade network centrality in Richmond (2019). The current paper pursues the nontraded output growth risk on the basic intuition that such a risk is internalized by, and impactful to, the host country's asset prices. We demonstrate that nontraded output growth risk is a structural factor that is priced in interest rates, exchange rates, and currency premia. Lustig et al. (2011) show that the cross-sectional variation in carry trade returns is driven by heterogeneous exposures of currencies to a common risk factor (and a country-specific risk factor). Our nontraded and traded output growth risks can be cast in their general factor pricing framework, and provide an economic interpretation, i.e., the tradability of consumption goods, for their reduced-form factors. The pricing effects of output nontradability have been studied in early work by Stulz (1987), Stockman and Dellas (1989), Backus and Smith (1993), and Zapatero (1995), and the pricing effects of trade costs by Dumas (1992) and Sercu et al. (1995). Our empirical analysis builds on this literature to merge and match various OECD output, export and import data series and revisions to produce a comprehensive partial tradability index for goods at the most detailed industry level. No-arbitrage factor pricing and strategies of currency markets are investigated in many papers, including Brunnermeier et al. (2009) and Burnside et al. (2011) on skewness risks, Lustig et al. (2011) on SDF paradigm, Menkhoff et al. (2012) on volatility risks, Mueller et al. (2017) on the cross-sectional dispersion of exchange rate correlations, Maurer et al. (2017) on principal component (PCA) factors. Our paper pursues currency portfolio strategies based on nontraded output growth risks. We demonstrate empirically that the portfolios' returns sort with the risk exposure, and the implied price of nontraded output growth risk (i.e., factor price) is significant in these portfolios.

Our paper is most closely related to Hassan (2013), who demonstrates the role of economic size in interest rates and international asset pricing. Nontraded growth risks build upon the economic size factor, and extends it in several aspects. Basically, the difference in the pricing of risks and assets in different countries originates from difference in their numeraires, i.e., country-specific consumption baskets. In the difference with Hassan (2013), who studies the SDF associated with the numeraire of the common traded consumption good, we investigate the country-specific SDF associated with the numeraire of the country's consumption basket. As a result, our risk premia reveal the pricing of nontraded output risk from the local investors' perspective, for whom the nontraded output risk matters the most. Nontraded consumption goods, which are country-specific by definition, are the key component that differentiates the consumption baskets, or numeraires, of different countries. In settings of a common traded good but no nontraded goods, exchange rates are one,[3] and interest rates are identical in all countries regardless of differences in their economic sizes. This is because bonds in all countries deliver the same consumption basket, hence offer the same hedging power and have the same valuation (i.e., price and interest rate). On the other hand, in the presence of nontraded goods, exchange rates differ from one and different countries have different interest rates even when they have the same economic size. This indicates that in consumption-based settings, economic sizes influence asset pricing only via their coupling with nontraded output growth risks,

[3]This is because countries have the same consumption basket (i.e., the numeraire), which is a unit of traded good. When there are multiple traded goods, exchange rates can differ from one when countries have heterogeneous tastes for traded goods (resulting in different consumption baskets for different countries). The nontraded goods offer one of simplest frameworks to exemplify such heterogeneous tastes.

whereas these risks also affect asset pricing on their own. Empirically, currencies that have similar (e.g., funding) characteristics in carry trades can be associated with economies of diverse sizes (e.g., Switzerland versus Japan or USA), indicating that economic sizes alone do not explain currency premia. In the nontraded consumption framework, the difference in sizes are offset by the difference in nontraded output risks in these economies so that short positions on these currencies offer similar return characteristics.

The paper is structured as follows. Section 2 presents a basic international asset pricing model with traded and nontraded goods, trade costs, and testable asset pricing implications. Section 3 presents empirical tests on the pricing effects of nontraded output growth risks in interest rate and currency markets. Section 4 concludes. Appendices A and B describe sources and treatment of data. Appendix C presents technical details of trade costs, and Appendix D presents other derivations omitted in the main text. Online Appendix E discusses extensions with multiple traded goods, arbitrary trade configuration and incomplete financial markets.

2 Basic Model

The basic model of the world endowment economy consists of K countries. Each country engages in the international trade of a common consumption good (i.e., traded good), and also consumes a country-specific good that can be consumed only in that country (i.e., nontraded good). Generalized versions of the model consider good-specific transportation (trade) costs that endogenize the cross section of equilibrium consumptions of these goods in complete and incomplete markets (Section 2.1 and Appendix C). This parsimonious modeling approach aims to provide intuitions for testable implications on the equilibrium relationship between nontraded output risks and asset returns (Section 3). Our endowment economy framework abstracts from production and calibration aspects of traded and nontraded outputs and their trade costs.

Specifically, we take outputs in each country as the given country-specific endowments $\{\Delta_T^H, \Delta_N^H\}$, which follow diffusion processes

$$d\log\Delta_T^H = \mu_T^H dt + \sigma_T^H dZ_T^H; \quad d\log\Delta_N^H = \mu_N^H dt + \sigma_N^H dZ_N^H; \quad H \in \{1, \dots, K\}. \tag{1}$$

Throughout, the superscript H denotes the country, subscripts T, N the traded and nontraded goods, and Z_T^H and Z_N^H the standard (possibly multi-dimensional) Brownian motions characterizing the country-specific supply shocks of the traded and nontraded sectors. All quantities in the model are real. For simplicity, we also omit time index t when possible. We assume that financial markets are complete.[4] Each country is characterized by a representative agent whose utility is a function of both traded and nontraded per-capita consumptions

$$U^H(C^H, t) = e^{-\rho t}\frac{(C^H)^{1-\gamma}}{1-\gamma} = e^{-\rho t}\frac{1}{1-\gamma}\left[\omega_T (C_T^H)^{1-\epsilon} + \omega_N (C_N^H)^{1-\epsilon}\right]^{\frac{1-\gamma}{1-\epsilon}}; \quad \omega_T + \omega_N = 1, \quad (2)$$

where ρ denotes the time discount factor. This utility features constant intertemporal elasticity of consumption of $\frac{1}{\gamma}$, and constant elasticity of substitution between traded and nontraded goods of $\frac{1}{\epsilon}$. They satisfy the conditions $\gamma > 0$, $\epsilon > 0$. Countries may have different tastes $\{\omega_T, \omega_N\}$ for traded and non traded goods. For a simple exposition, we first consider no trading costs in goods markets so that traded good outputs from all countries are pooled into the aggregate traded endowment

$$\Delta_T \equiv \sum_{H=1}^{K} \Delta_T^H; \qquad d\log\Delta_T \equiv \mu_T dt + \sigma_T dZ_T, \quad (3)$$

that determines the traded good supply dynamics.[5] We later address trading costs and relate them to the degree of tradability of goods in data.

In equilibrium, complete financial markets and the absence of trade costs imply that countries are able to perfectly share traded output risks, resulting in equal marginal utilities of the traded consumption good across countries

$$\Lambda^H \frac{\partial U^H}{\partial C_T^H} = \Lambda^F \frac{\partial U^F}{\partial C_T^F} \equiv M_T, \qquad \forall H, F \in \{1, \ldots, K\}, \quad (4)$$

where $\{\Lambda^I\}$, $I \in \{1, \ldots, K\}$, denote countries' Pareto weights, and characterize the economic sizes of countries. These weights can only be determined up to a common multiplicative constant in equilibrium, so only their relative sizes matter.[6] We log-linearize these nonlinear first-order conditions (FOCs) to obtain an approximate but intuitive solution for an analysis of equilibrium.

[4] Appendix E.2 generalizes the analysis to incomplete market settings.

[5] Note that moments μ_T and σ_T are functions of the country-specific moments μ_T^I and σ_T^I in (1) due to the the aggregation of traded endowments. Because only aggregated traded endowment Δ_T and its moments appear explicitly in equilibrium quantities, our discussion mainly concerns the specification of the traded endowment (3) at the aggregate level. Country-specific traded endowment risks are subject to diversification in the aggregation process of traded endowment. Through this process, their specifications (1) contribute to, and are absorbed in, the sizes of economies in our analysis.

[6] Therefore, weights $\{\Lambda^I\}$ can be measured in any common numeraire, such as U.S. dollar or any other currency. In the empirical part, we follow the literature to use countries' GDPs to proxy for these weights, where GDPs are measured in a common numeraire (USD).

Equilibrium traded consumptions: Let the lower-case letters always denote the respective log quantities; $c \equiv \log C$, $\delta_T \equiv \log \Delta_T$, $\delta_N \equiv \log \Delta_N$. The equilibrium log (per-capita) traded consumption is (Appendix D)

$$c_T^H = \delta_T + \frac{1}{\gamma\omega_T + \epsilon\omega_N} \left\{ -\rho t - (\gamma - \epsilon)\omega_N \left(\left[1 - \frac{\Lambda^H}{\Lambda} \right] \delta_N^H - \sum_{F \neq H}^{K} \frac{\Lambda^F}{\Lambda} \delta_N^F \right) \right\}, \tag{5}$$

where we recall that δ_T is the log *aggregate* traded output. $\Lambda \equiv \sum_{H=1}^{K} \Lambda^K$ is a measure of the global GDP, therefore $\frac{\Lambda^H}{\Lambda}$ the relative GDP size of countries. A country's equilibrium traded consumption is influenced by the substitution of nontraded-traded consumptions and international trades. H adjusts its traded consumption c_T^H to (i) respond to changes in aggregate traded endowment δ_T, (ii) substitute for changes in H's nontraded output δ_N^H, and (iii) respond to changes in trade partner F's traded consumption demand due to F's nontraded endowment shock. The aggregate, but not country-specific, log traded endowment δ_T affects the log traded consumption of every country uniformly as a result of frictionless trading and perfect sharing of the traded output risk.

Hassan (2013) analyzes the consumption allocation (5) with an emphasis on the role of economic sizes of countries. We focus on *nontraded* output growth risks and their interactions with economic sizes. While the foreign trade partner's nontraded output impact on c_T^H (5) is enhanced by the foreign country's size (term $\frac{\Lambda^F}{\Lambda}$), country H's own nontraded output impact is subdued by its size (term $1 - \frac{\Lambda^H}{\Lambda}$). Because fluctuations in *per-capita* traded consumption of a larger trade partner F project a larger impact on the common marginal utility, the size Λ^F amplifies F's nontraded shock impact on the rest of the world. In contrast, a larger home country H requires a larger traded consumption adjustment, i.e., a more challenging transaction with its trade partners.[7]

Stochastic discount factors (SDFs): In consumption-based settings, the SDF of a country H is the marginal utility of the country-specific consumption basket, or the lowest-cost consumption bundle that delivers one unit of H's utility, $M^H = \frac{\partial U^H}{\partial C^H} = e^{-\rho t}(C^H)^{-\gamma}$. The home-specific log SDF, $m^H \equiv \log M^H$, is (Appendix D)

$$m^H = -\rho t - \gamma\omega_T\delta_T - \gamma\omega_N \left[\delta_N^H - \alpha(\gamma - \epsilon)\omega_T \left(1 - \frac{\Lambda^H}{\Lambda} \right) \delta_N^H + \alpha(\gamma - \epsilon)\omega_T \sum_{F \neq H}^{K} \frac{\Lambda^F}{\Lambda} \delta_N^F \right], \tag{6}$$

[7] This adjustment arises via the substitution between nontraded and traded consumptions. Quantitatively, when $\gamma > \epsilon$ the elasticity of substitution $\frac{1}{\epsilon}$ dominates that of intertemporal substitution $\frac{1}{\gamma}$. Investors are primarily concerned with smoothing consumption over time by adjusting their traded-nontraded consumption composition. Traded consumption c_T^H responds strongly to nontraded supply shocks, increases with δ_N^F and decreases with δ_N^H.

where $\alpha \equiv (\gamma\omega_T + \epsilon\omega_N)^{-1}$ is a weighted elasticity of substitution. Note that M^H characterizes the asset pricing from the home investors' perspectives and in the numeraire of home currency (i.e., home consumption basket). The country-specific SDF conceptually differs from the common marginal utility of the traded consumption M_T (4), which prices assets in the numeraire of the common traded consumption good. This distinction is important because a financial asset has different risk-return tradeoff interpretations (i.e., risk premia) in different pricing numeraires. For an illustration, since a country's risk-free bond pays one country-specific consumption basket at maturity, the country-specific risk-free rate r_H is intrinsically associated with SDF M_H. In this respect, our analysis differs from that of Hassan (2013), which concerns SDF M_T and the associated risk-free rate that are based on the numeraire of the traded consumption.

We observe that the home SDF moves with the home nontraded endowment δ_N^H in two fashions. Indirectly (the second term inside the square brackets in (6)), although H consumes its entire nontraded endowment, H still is able to partially substitute and mitigate its nontraded output shocks by adjusting its traded good's intake via international trades. The effectiveness of this mitigation is lower for a larger home economy, who needs to execute a larger international trade of traded goods to substitute for fluctuations in the home nontraded output. Directly and importantly (the first term inside the square brackets), movements in the nontraded output δ_N^H are fully reflected in the log SDF m^H. This direct effect arises from the internalization of nontraded output by the host country, and hence does not involve country's size. The nontraded output growth risk goes beyond, and decouples from, economy's size in pricing assets in this direct effect. Combining direct and indirect effects, the nontraded output risk of a larger economy is more impactful to the economy's SDF because the mitigation of this risk (indirect effect) is more limited via international trades.

Proposition 1 *In the current setting of the world economy, although the nontraded output shock of a country is priced by all of its trade partner economies, the home nontraded output risk always influences the home SDF m^H more than it influences the foreign SDF m^H; $\left|\frac{\partial m^H}{\partial \delta_N^H}\right| > \left|\frac{\partial m^F}{\partial \delta_N^H}\right|$.*

That is, the coefficient associated with δ_N^H is invariably larger in m^H than in any other m^F for all size configurations. Below, we investigate the implications of this result on interest rates and currency premia.

2.1 Interest Rates

A country-specific interest rate is the instantaneous return on the country-specific bond, which is priced by the country-specific SDF (6). Therefore, the interest rate is given by the drift term of this SDF,[8]

$$\begin{aligned} r^H &= \rho + \gamma\omega_T\mu_T - \frac{1}{2}\gamma^2\omega_T^2\sigma_T^2 + \alpha\gamma(\gamma-\epsilon)\omega_T\omega_N\sum_{F=1}^{K}\frac{\Lambda^F}{\Lambda}\mu_N^F - \frac{1}{2}\alpha^2\gamma^2(\gamma-\epsilon)^2\omega_T^2\omega_N^2\sum_{F=1}^{K}\frac{(\Lambda^F)^2}{(\Lambda)^2}(\sigma_N^F)^2 \\ &+ \alpha\gamma\epsilon\omega_N\mu_N^H - \frac{1}{2}\alpha^2\gamma^2\epsilon^2\omega_N^2(\sigma_N^H)^2 - \alpha^2\gamma^2\epsilon(\gamma-\epsilon)\omega_T\omega_N^2\frac{\Lambda^H}{\Lambda}(\sigma_N^H)^2. \end{aligned} \tag{7}$$

The expected endowment growth rates μ's contribute to increase risk-free rates via the intertemporal consumption smoothing effect. Endowment growth volatilities σ's act to suppress risk-free rates via precautionary savings effect. Terms containing coefficients $(\gamma-\epsilon)\omega_T$ arise from traded consumption sharing and adjusting, wherein ω_T characterizes investors' affection for the traded good and $(\gamma-\epsilon)$ the nontraded shock spill-over to the traded sector by the substitution of two consumption goods. We observe that nontraded output risks drive the interest rate differential,[9]

$$r^H - r^F = \alpha\gamma\epsilon\omega_N\Delta\mu_N - \frac{1}{2}\alpha^2\gamma^2\epsilon^2\omega_N^2 - \alpha^2\gamma^2\epsilon(\gamma-\epsilon)\omega_T\omega_N^2\left(\frac{\Lambda^H}{\Lambda}(\sigma_N^H)^2 - \frac{\Lambda^F}{\Lambda}(\sigma_N^F)^2\right), \tag{8}$$

The home nontraded output risk $(\sigma_N^H)^2$ impacts the home interest rate r_H both on its own and on its coupling with the host economy's size $\frac{\Lambda^H}{\Lambda}$. The overall effect is that, the home nontraded risk is more severe, making home-specific bond more valuable and lowering the home interest rate, when the home economy is larger. This is because a larger home economy finds it more difficult to mitigate its own nontraded output risk, an economic feature also found in the country-specific SDF m^H (6).

The analysis above indicates very different implications of traded and nontraded growth risks on risk-free rates. We formulate a testable hypothesis highlighting this difference.

Hypothesis 1 *All else being equal, the impact of country-specific nontraded output growth risk on home interest rate dominates that of the country-specific traded output growth risk.*

[8] That is, $r^H = \frac{1}{dt}\left(-E_t\left[dm^H\right] - \frac{1}{2}\mathrm{Var}_t\left[(dm^H)^2\right]\right)$ (6). For simplicity, we assume that country-specific nontraded outputs are pairwise uncorrelated, and uncorrelated with aggregate traded outputs (extensions are relaxed in Appendix E).

[9] The first five terms (i.e., all terms in the first line of (7)) of risk-free rates are identical across countries. In (8), $\Delta\mu_N$ and $\Delta(\sigma_N)^2$ denote respectively the differential of mean and variance of nontraded output growths across economies H and F.

Section 3.2 present actual tests and supportive evidences of this hypothesis. Intuitively, the diversification of traded output risks gives rise to the hypothesis. To see this, we concentrate on the contributions of country-specific traded output volatilities σ_T^H to the interest rate (7) (omitting terms unrelated to these volatilities),

$$r^H = \# - \frac{1}{2}\gamma^2\omega_T^2\sigma_T^2 = \# - \frac{1}{2}\gamma^2\omega_T^2\frac{1}{dt}\left(\sum_{H=1}^{K}\frac{\Delta_T^H}{\Delta_T}\sigma_T^H dZ_T^H\right)^2. \tag{9}$$

Clearly, country-specific traded shocks dZ_T^H are weakened by the share of a country's traded output in the world $\frac{\Delta_T^H}{\Delta_T}$, because country-specific traded outputs are pooled into the aggregate global output thanks to the tradability of these outputs.[10]

Costly Trades and Partial Tradability

The previous section's results are derived based on two assumptions, namely, goods are either perfectly traded or nontraded, and trades are frictionless. Consequently, traded output growth risks are aggregated and diversified across borders, giving rise to Hypothesis 1. Trade costs relax both of these simplifications, hence also presents a modeling aspect to address the exogenous setup of traded versus nontraded good outputs of endowment economies. In particular, the *tradability* degree of a consumption good can be non-perfect, and is regulated by the trade cost. A good can become a nontraded (traded) good when trade cost is sufficiently high (low). The tradability is the key to bringing our model to the data in Section 3.2.

To model the frictions in trades, we adopt the "iceberg transport cost" of Samuelson (1954), Dumas (1992) and Sercu et al. (1995). In this approach, the commodity trade is not perfect because only a fraction of $\frac{1}{1+\theta}$ of the original traded good that leaves the exporting country arrives at the importing country, and the remainder "melts" along the way as a result of trade friction. Parameter θ regulates the amount of good being traded (import and export) between countries, and hence effectively, the tradability of goods. For simplicity, we first consider a single good traded among two countries $\{H, F\}$ of similar sizes.[11] We relegate technical derivations of all results concerning trade costs to Appendix C.

Depending on the country-specific supplies, the transportation cost, and risk aversion, there are

[10] Tian (2018) makes a related observation that a country's traded consumption growth should be less volatile than the country's traded output growth due to the diversification in the traded good market.

[11] We therefore drop the subscripts T, N. The modeling of various transportation costs for several goods is similar to that of a single good.

two regimes,

$$\text{No-trade regime:} \quad (1+\theta)^{-1} < \left(\frac{\Delta^H}{\Delta^F}\right)^{\gamma} < (1+\theta),$$
$$\text{Trade regime:} \quad \text{otherwise}$$

Clearly, either costly transport (large θ), or similar outputs ($\frac{\Delta^H}{\Delta^F} \approx 1$), or low risk aversion (small γ), discourages countries to trade to share risk, and thus enforces the commodity market no-trade. In this case, the good effectively becomes a nontraded good in both countries. A country's bond does not hedge others' shocks, and the risk-free rate solely reflects the country's own output risk. For each country, the nontraded output volatility is the only risk that matters here. In contrast with the no-trade regime, when friction is moderate and home and foreign outputs are sufficiently different, countries choose to trade to share output risks. As long as trades take place, marginal utilities are equalized across countries ($\frac{\partial U^H}{\partial C^H} = (1+\theta)\frac{\partial U^F}{\partial C^F}$), as are the interest rates. (in the current setting with a single good). Conditioning on trades taking place, the effect of precautionary savings on interest rates is similar to (9)

$$r^H = r^F = \# - \frac{1}{2}\gamma(\gamma+1)\frac{(1+\theta)^2(\Delta^H)^2(\sigma^H)^2 + (\Delta^F)^2(\sigma^F)^2}{[(1+\theta)\Delta^H + \Delta^F]^2}. \tag{10}$$

As the transport cost θ increases, interest rates become increasingly sensitive to home output shocks and decreasingly sensitive to foreign output shocks; $\frac{\partial^2 |r|}{\partial\theta\partial|(\sigma^H)^2|} > 0$, $\frac{\partial^2 |r|}{\partial\theta\partial|(\sigma^F)^2|} < 0$. These behaviors indicate that when shocks are of a more nontraded nature (i.e., θ increases), they matter more to the country's asset prices. From the importing country H's perspective, a surge in trade cost θ coincides with a reduction in trades as its imports C_F^H drop. At the same time, the impact of the country's own volatility σ^H on its interest rate r^H increases while the impact of foreign volatility σ^F on r^H decreases. Likewise, from the exporting country F's perspective, a surge in trade cost coincides with a boost in trades as its export $(1+\theta)C_H^F$ increases. At the same time, the impact of its own volatility σ^F on its interest rate r^F decreases, whereas the impact of partner's volatility σ^H on r^F increases. Overall, the variation in trade frictions implies a structural relationship between nontradability and domestic asset prices.

Hypothesis 2 *All else being equal, a country-specific output growth volatility impacts the home risk-free rate more when the output is less tradable.*

This is refined version of Hypothesis 1 that allows for partial tradability. Section 3.2 tests this hypothesis by employing different empirical measures of nontradability.

2.2 Carry Trade Returns

Let us consider the typical carry trade strategy of borrowing the home and lending the foreign currency at t, liquidating all positions at $t + dt$ from home investors' perspective. Let the spot exchange rate S_t be the amount of foreign currency that buys one unit of the home currency at t. In complete market settings, the exchange rate equals the SDF ratio, $S_t = \frac{M_t^H}{M_t^F}$. The realized and expected returns, in excess of the home interest rate, on this carry trade are

$$\begin{aligned} XR_{t+dt}^{-H,+F} &= \frac{1}{dt}\left[\frac{M_{t+dt}^F}{M_{t+dt}^H}(1+r_t^F dt)\frac{M_t^H}{M_t^F} - (1+r_t^H dt)\right], \\ E_t\left[XR_{t+dt}^{-H,+F}\right] &= -\frac{1}{dt}Cov_t\left[dm^H, dm^F - dm^H\right]. \end{aligned} \tag{11}$$

The consumption fluctuations contribute to the currency premium $E_t\left[XR_{t+dt}^{-H,+F}\right]$ precisely because they perturb both SDFs m^H, m^F via direct consumption and trade. Because traded shocks spread uniformly to all countries, they do not affect exchange rates, and are not priced in the carry trades. This leaves fluctuations in nontraded outputs as the sole source of currency risk and return in the current setting. Indeed, the log exchange rate is obtained from (6)

$$d\log S_t = dm^H - dm^F = \#dt + \gamma\alpha\epsilon\omega_N\left(\sigma_N^H dZ_N^H - \sigma_N^F dZ_N^F\right). \tag{12}$$

On one hand, an adverse foreign nontraded shock $dZ_N^F < 0$ makes F's nontraded good scarce and suppresses the real exchange rate S (i.e., foreign currency appreciates), and therefore $m^F - m^H$ surges (see also Proposition 1). On the other hand, $dZ_N^F < 0$ also forces F to consume more and H to consume less traded goods, and m^H surges. That is, the long bet on the foreign currency pays off well when home investors highly value consumption. This carry trade strategy hence is a *hedge* against the foreign nontraded risk. All else being equal, the expected return $E_t\left[XR^{-H,+F}\right]$ is lower when the foreign nontraded risk is more significant in equilibrium.

In contrast, an adverse home nontraded shock $dZ_N^H < 0$ directly boosts m^H. Moreover, H demands more traded goods to substitute for its loss in nontraded goods, leaving F with less traded consumptions. Hence, m^F increases but by a lesser extent than m^H does. Consequently, $m^F - m^H$ drops because the real exchange rate S increases (i.e., home currency appreciates). That is, the long bet on the foreign currency pays off poorly when home investors highly value consumption. This carry trade strategy hence is *not* a hedge against the home nontraded risk. All else being equal, the expected return $E_t\left[XR^{-H,+F}\right]$ is higher when the home nontraded risk is more significant. The overall expected profit (or loss) of the carry trade is determined by whether home (or foreign) nontraded risk dominates, as seen quantitatively in the following result.

Proposition 2 *The expected carry trade excess return to U.S. investors is*

$$E_t\left[XR_{t+dt}^{-H,+F}\right] = \alpha^2\gamma^2\epsilon\omega_N^2 \left\{\left[\epsilon + (\gamma-\epsilon)\omega_T\frac{\Lambda^H}{\Lambda}\right](\sigma_N^H)^2 - (\gamma-\epsilon)\omega_T\frac{\Lambda^F}{\Lambda}(\sigma_N^F)^2\right\}, \qquad (13)$$

where $\alpha \equiv (\gamma\omega_T + \epsilon\omega_N)^{-1}$ *is a weighted elasticity of consumption substitution. Consequently, the carry trade strategy offers an expected profit when the home nontraded output risk coupled with the home size dominates,*

$$\left[\epsilon + (\gamma-\epsilon)\omega_T\frac{\Lambda^H}{\Lambda}\right](\sigma_N^H)^2 > (\gamma-\epsilon)\omega_T\frac{\Lambda^F}{\Lambda}(\sigma_N^F)^2.$$

The carry trade is a good (bad) hedge against the foreign (home) nontraded output growth risk. When home nontraded risk dominates, $(\sigma_N^H)^2 \gg (\sigma_N^F)^2$, this strategy is risky and offers high expected returns $E_t\left[XR_{t+dt}^{-H,+F}\right] > 0$, and vice versa. However, when $(\gamma-\epsilon)\omega_T$ is positive but small, investors are not enthusiastic about substituting nontraded for traded consumption goods. This weakens the trade effect, makes home nontraded output risk even worse to home investors, and compensates them with higher expected returns.

Our analysis not only provides a risk-based explanation for widely-practiced carry trade strategies, it also suggests the following currency bet based on the nontradability aspects of output growth risk revealed in (13),

Hypothesis 3 *Borrowing currencies of economies with volatile nontraded sectors and large sizes while lending currencies of economies with stable nontraded sectors and small sizes generate positive expected returns.*

We examine empirically the merits of this macro-based strategy in Section 3.3. When the home country has volatile nontraded sector (σ_N^H large), home risk-free bonds are valuable as a safe asset, and the home interest rates is low (r^H small). At the same time, expected carry trade returns are high because these strategies are exposed to the significant home nontraded risk. When the foreign nontraded sector is stable (σ_N^F small), the foreign interest rate is high (r^F large), *and* expected carry trade returns to home investors are also high (Proposition 2). All in all, nontraded output risks, originated from either home or abroad, rationalize the profitability of currency strategies of borrowing currencies of low interest rates and lending ones of high interest rates, i.e., the observed violation of the uncovered interest rate parity (UIP).

Linear Factor Analysis: Theory

The SDF (6) shows that country-specific traded and nontraded shocks are priced differently. In particular, country-specific traded shocks are absorbed into the aggregated traded endowment δ_T, while country-specific nontraded shocks to δ_{NT}^H contribute to m^H (6) either by themselves or through the coupling with economic sizes. This intuitive finding motivates the construction of a simple linear-factor pricing model in which the risk factors are traded and nontraded consumption growths

$$f_T^H = \frac{dC_T^H}{C_T^H}; \qquad\qquad f_N^H = \frac{dC_N^H}{C_N^H}.$$

We employ carry trade portfolios as test assets to evaluate this pricing model conceptually (below) and empirically (Section 3.3.2).

The factor analysis starts with the *unconditional* consumption-based Euler equation for this carry trade return,[12]

$$E\left[\frac{M_{t+dt}^H}{M_t^H} XR_{t+dt}^{-H,+F}\right] = 0 \Longrightarrow E\left[XR_{t+dt}^{-H,+F}\right] = -\frac{1}{dt} Cov\left[dm_{t+dt}^H, XR_{t+dt}^{-H,+F}\right], \tag{14}$$

where $dm_{t+dt}^H \equiv \log M_{t+dt}^H - \log M_t^H$ (6). Because the home consumption is made of both traded and nontraded components, the log-linearized SDF (6) immediately implicates a linear two-factor

[12]The unconditional Euler equation (14) follows from its conditional version $E_t\left[\frac{M_{t+dt}^H}{M_t^H} XR_{t+dt}^{-H,+F}\right] = 0$ by taking the unconditional expectation of both sides (applying the law of iterated expectation), $E\left[\frac{M_{t+dt}^H}{M_t^H} XR_{t+dt}^{-H,+F}\right] = 0$, or

$$E\left[\frac{M_{t+dt}^H}{M_t^H}\right] E\left[XR_{t+dt}^{-H,+F}\right] = -Cov\left[\frac{M_{t+dt}^H}{M_t^H}, XR_{t+dt}^{-H,+F}\right].$$

The carry trade excess return is of order $\mathcal{O}(dt, dZ_{t+dt})$: $XR_{t+dt}^{-H,+F} = \#dt + \#dZ_{t+dt}$, and the SDF growth is of order $1 + \mathcal{O}(dt, dZ_{t+dt})$: $\frac{M_{t+dt}^H}{M_t^H} = 1 + \#dt + \log\frac{M_{t+dt}^H}{M_t^H} \equiv 1 + \#dt + dm_{t+dt}^H$. Therefore, up to the order $\mathcal{O}(dt)$, the left-hand side of above equation is $E\left[XR_{t+dt}^{-H,+F}\right]$, while the right-hand side is $-Cov\left[dm_{t+dt}^H, XR_{t+dt}^{-H,+F}\right]$, implying (14).

pricing model of the carry trade from the home perspective

$$E\left[XR_{t+dt}^{-H,+F}\right] = -Cov\left[b_T f_{T,t+dt}^H + b_N f_{N,t+dt}^H, XR_{t+dt}^{-H,+F}\right] \tag{15}$$

$$\begin{bmatrix} b_T \\ b_N \end{bmatrix} = \begin{bmatrix} -\gamma\omega_T \\ -\gamma\omega_N \end{bmatrix}; \quad \begin{bmatrix} f_T^H \\ f_N^H \end{bmatrix} = \begin{bmatrix} \frac{dC_T^H}{C_T^H} \\ \frac{dC_N^H}{C_N^H} \end{bmatrix} = \begin{bmatrix} d\delta_T - \alpha(\gamma-\epsilon)\omega_N\left(d\delta_N^H - \sum_{F=1}^K \frac{\Lambda^F}{\Lambda} d\delta_N^F\right) \\ d\delta_N^H \end{bmatrix},$$

where $\{f_T^H, f_N^H\}$ are risk factors, and $\{b_T, b_N\}$ the associated loadings on these factors. This linear-factor model prices risks from the home investors' perspective, and can also be cast in its beta-pricing version

$$E\left[XR_{t+dt}^{-H,+F}\right] = \lambda_T^H \beta_T^{H,F} + \lambda_N^H \beta_N^{H,F}, \tag{16}$$

$$\begin{bmatrix} \lambda_T^H \\ \lambda_N^H \end{bmatrix} \equiv \left[Cov(\vec{f}^H, \vec{f}^H)\right] \begin{bmatrix} -b_T \\ -b_N \end{bmatrix}; \quad \begin{bmatrix} \beta_T^{H,F} \\ \beta_N^{H,F} \end{bmatrix} \equiv \left[Cov(\vec{f}^H, \vec{f}^H)\right]^{-1} \begin{bmatrix} Cov(f_T^H, XR^{-H,+F}) \\ Cov(f_N^H, XR^{-H,+F}) \end{bmatrix},$$

where $\left[Cov(\vec{f}^H, \vec{f}^H)\right]$ denotes the 2×2 variance-covariance matrix of the factors $\{f_T^H, f_N^H\}$. Slope coefficients β (of returns regressed on the risk factors) quantify the exposures of investment strategies to the two risk factors. Factor prices $\{\lambda_T^H, \lambda_N^H\}$ are the rewards (i.e., expected returns) to bear one notional unit of corresponding risk (i.e., as if $\beta = 1$). Therefore, a positive factor price indicates that the associated factor presents a risk to home investors. The substitution of factors f's and loadings b's from (15) into (16) yields the following testable result concerning the nontraded factor price.

Proposition 3 *The factor price associated with nontraded consumption growth risk is unambiguously positive*

$$\lambda_N^H = \alpha\gamma\omega_N\left[\epsilon + (\gamma-\epsilon)\omega_T\frac{\Lambda^H}{\Lambda}\right](\sigma_N^H)^2 > 0 \quad \forall H. \tag{17}$$

That is, fluctuations in the domestic nontraded consumption growth always pose as a risk to home investors in all countries.

The nontraded factor price λ_N^H (17) is country-specific. It involves only the volatility σ_N^H because the nontraded output can only be consumed domestically. As smaller economies are able to manage this risk better by more flexibly adjusting their smaller traded consumptions, this risk is less costly for them. Indeed, λ_N^H increases with economy's size Λ^H (17).

Similarly, substituting (15) into (16) yields the home factor price of the traded consumption growth risk

$$\lambda_T^H = \gamma\omega_T(\sigma_T)^2 + \alpha^2\gamma(\gamma-\epsilon)^2\omega_T\omega_N^2 \sum_{F\neq H}^{K} \frac{(\Lambda^F)^2}{(\Lambda)^2}(\sigma_N^F)^2 - \alpha^2\gamma(\gamma-\epsilon)\omega_N^2\left(1-\frac{\Lambda^H}{\Lambda}\right)\left[\epsilon+(\gamma-\epsilon)\omega_T\frac{\Lambda^H}{\Lambda}\right](\sigma_N^H)^2. \tag{18}$$

The traded factor price λ_T^H is also country-specific. Nontraded out risks are always enhanced by the associated country's size in λ_T^H because these risks matter to the home traded consumption via the substitution of traded and nontraded consumptions. In difference with λ_N^H (Proposition 3), the sign of λ_T^H depends on the relative magnitudes of global traded and nontraded output risks (enhanced by countries' sizes). Conceptually, the nontraded risk factor is a priced factor in currency markets that also explains the interest rate differential across economies (Hypothesis 2) underling the high-minus-low (HML, or long-short) carry trade strategy in FX markets. These characteristics fit the narrative of the slope factor found in a reduced-form approach by Lustig et al. (2011) (and denoted HML_{FX} therein) that is key to the pricing of currency risks. Section 3.3.2 presents the estimates of factor prices for the U.S. and further analysis.

3 Empirical Results

The principal assertion of this paper, motivated by theoretical considerations in preceding sections, is that fluctuations in nontraded output growths are an important risk factor determining asset prices in international markets. This section investigates this assertion empirically and provides supportive evidence. We implement various tests on interest rates and carry trade returns for OECD countries plus Eurozone (i.e., Economic and Monetary Union, available after 1998). These are more developed economies, whose data series tend to be more complete and of higher quality.[13] Since the model is real while data are nominal, we first factor out the inflation component from empirical quantities, before using them in tests of model's implications. All employed data series are cited in double quotes, and their original sources and other details are listed in the data appendix. Since data at the industry level (ISIC-4) are more elaborate than the outputs in the model, our empirical analyses are motivated by and follow the model's intuitions but do not calibrate the model itself.

[13] In our notation, before the German reunification in 1990 (and including that year), the Federal Republic of Germany (FRG) is referred to as West Germany. From 1991 onward, the (reunified) Federal Republic of Germany is referred to as Germany.

3.1 Stylized Facts Concerning Nontraded Output Risk

We identify "construction" and "services" as nontraded sectors in all countries, following the standard classification in the literature (see, e.g., Stockman and Tesar (1995)). Components of services sectors include the following International Standard Industrial Classification of All Economic Activities Revision 4 (ISIC 4) codes: (F) Construction; (G) Wholesale, retail trade and repair; (H) Transportation and storage; (I) Accommodation and food service activities; (J) Information and communication; (K) Financial and insurance activities; (L) Real estate activities; (M) Professional, scientific and technical activities; (N) Administrative and support service activities; (O) Public administration, defence, social security; (P) Education; (Q) Human health and social work activities; (R) Arts, entertainment and recreation; (S) Other service activities; (T) Activities of households as employers; and (U) Activities of extraterritorial organizations and bodies.

To have an idea about the size of nontraded sectors in the economies worldwide, Figure 1 plots the ratio of nontraded output over real GDP, averaged over the period 1970-2016, for all OECD countries plus Eurozone. Output data are from "Aggregate National Accounts: Gross domestic product", and services output are computed as the sum of outputs in all ISIC 4 codes F-U. The figure shows that nontraded outputs constitute a substantial fraction of the total GDP in all OECD countries, ranging from 0.45 (New Zealand) to 0.67 (U.S.) to 0.80 (Luxembourg). Among others, this figure thus re-documents a known fact that services sectors carry a significant weight of the U.S. economy.

To justify the identification of services as a nontraded sector, Table 1 lists the country-specific tradability and size of different service categories for a representative set of 13 OECD countries (see data appendix for classification details). Tradabilities and sizes are averaged over the period 1970-2016. The country-tradability of services is (one half of) the ratio of total exports and imports over the total output of these services by the country (see (21)). The economic size of services is the ratio of total domestic output of these services over the country's GDP. All import, export and output data for each service category is from "OECD Structural Analysis" (STAN), combined with "Trade by Enterprise Characteristics" (TEC) database.[14] Country total GDP is from "Aggregate National Accounts: Gross domestic product". The table shows that, whereas the tradabilities and sizes of the same services vary considerably across OECD economies, their tradabilities are indeed

[14]STAN 3 (up to 2009) has both trade and output data, STAN 4 (from 2009) has only output data, trade data is moved to TEC. Further information on the matching between STAN 3, STAN 4 and TEC data are provided in the data appendix.

small (in the order of few percentage points, and rarely exceeding 15%). In particular, service codes J-N (including information, communication, financial services, and other business activities) are a substantial part of GDP in all countries (ranging from 16.9% for Australia to 46.29% for Luxembourg), yet their tradabilities are very low (ranging from 0.01% for Australia to 1.52% for United Kingdom). Similarly, Table 2 lists the 15 major industries in the U.S., along with their two measures of tradability. The U.S.-specific tradability of an industry is computed using country-specific tradability (21) for the U.S. In the determination of OECD tradability (see (20)), export, import and output are OECD-aggregate quantities. These industry-level macro series are from the STAN and TEC databases. Table 2 shows a marked difference between the tradability of the first 9 industries (manufacturing sector) and the next 6 industries (service sector), which justifies the classification of traded and nontraded goods adopted in the literature as well as in the current paper. The table also shows that country-specific tradabilities do not necessarily and quantitatively coincide with their OECD counterparts because countries are heterogeneous in their consumption and production. For the sake of robustness, our tests presented in the next section will employ both of these tradability measures.

To have a sense of the level of nontraded output risk across countries, Figure 4 plots the volatility of per-capita nontraded output growth for each OECD country. The volatility is computed as the standard deviation of these per-capita nontraded output growth series over the entire period of 1970-2016. Per-capita quantities are computed using the population data from OECD "Aggregate National Account". This figure shows that the degree of fluctuations of nontraded output varies widely across OECD countries. In particular, Switzerland is a relatively small economy among OECD countries, yet its per-capita nontraded output growth is substantially more volatile than other countries (see also Figure 2). While our theoretical consideration (Section 2) indicates that nontraded output growth risks and sizes of economies are important for asset prices, it remains to see how these two factors combined contribute empirically in the cross section of OECD countries.

To have a sense of the level of trade "openness" of OECD countries, Figure 5 plots the ratio of each country's total exports and imports over its GDP (see also (19)), averaged over the period 1970-2016. The figure shows that trade openness is markedly heterogeneous across OECD countries, ranging from 0.19 for the U.S. to 2.21 for Luxembourg. It is known that this ratio can be biased downward for larger economies, and hence a low value of the openness for a country does not necessarily imply high (tariff or non-tariff) obstacles to foreign trade. Rather, the low value of the openness can be a measure of either a weak reliance of domestic producers on foreign supplies and markets or a country's geographic remoteness from potential trading partners. Any of these possible causes are consistent with our notion that the output growth risk of the less open economies is internalized by home countries to a larger extent.

3.2 Interest Rates

In reality, no goods are either perfectly nontraded or perfectly traded. Even if some goods were, macro output series are inevitably subject to measurement errors. Furthermore, costs in trades also affect the structural relation between nontraded output risk and asset prices. In this section, we investigate the empirical relationship between nontraded output volatility and the level of real interest rate across OECD countries, taking into account these practical regularities. We also control for the size of economies to focus on the output risk of various tradability degrees and its effect on real interest rates. Specifically, we devise four tests based on the various classifications of nontradability, in order of increasing sophistication. These regression-based tests involve (i) the closedness of an economy, (ii) the brute-force cutoff dummy of nontradability at the industry level, (iii)the global nontradabilities at the industry level, and (iv) country-specific nontradabilities at the industry level, respectively.

3.2.1 Tests Using Countries' Trade Closedness

The hypothesis to be examined here is that when an economy is exposed more to international trades, its nontraded risk can be better mitigated through trades and the substitution between traded and nontraded consumption. This assertion is motivated by the structural model with trade frictions in a specific form of Hypotheses 1-2 (Section 2.1). The basic regression test of this relationship reads

$$r_t^H = \alpha + \beta_\sigma (\sigma_t^H)^2 + \beta_C \mathcal{C}_t^H + \beta_{\sigma C} (\sigma_t^H)^2 \mathcal{C}_t^H + \beta_x X_t^H + \epsilon_t^H,$$

where σ^H denotes the per-capita GDP growth volatility and X's the various control variables. We adopt the common definition of a country's trade openness $\mathcal{O}^H$ as trade-to-GDP ratio (trade being the sum of export and import), from which also follows the closedness $\mathcal{C}^H$

$$\mathcal{O}^H = \frac{\text{IM}^H + \text{EX}^H}{\text{GDP}^H}; \qquad \mathcal{C}^H = 1 - \frac{\text{IM}^H + \text{EX}^H}{\text{GDP}^H}. \tag{19}$$

Table 3 reports the results associated with this regression. Both national output data and import/export data are from OECD "Aggregate National Accounts". We compute the volatility of per-capita GDP growth over each of five non-overlapping decades (1970-1979, 1980-1989, 1990-1999, 2000-2009, 2010-2016), and the mean of interest rates (dependent variable) over exactly the same periods. Control variables include per-capita GDP mean growth, GDP size (or the ratio of countries' GDP over the aggregate GDP of OECD group), and inflation volatility.[15] The key observation from Table 3 is that the slope coefficients associated with the interaction term (variance $\times$ closedness) are always negative. These coefficients are statistically significant when we take into account the GDP growth, economy size, and inflation risk effects. In particular, the control for the annualized mean of growth rate of per-capita real GDP (the variable "Growth mean" in Table 3) is important because this growth affects the interest rate (7) through the intertemporal consumption smoothing desire of investors, an effect that is outside of the nontraded output risk channel. This negative sign of the interaction term coefficients is consistent with the model's central economic rationale that, all else being equal, when a country is less open to trade, the country's output shocks tend to be more internalized and have stronger impacts on lowering country's real interest rate through the precautionary savings mechanism.

3.2.2 Tests Using Multiple Industry Outputs and Nontradability Dummies

Another form of Hypotheses 1-2 (Section 2.1) to be examined in this section is as follows. Controlling for anything else, a country's output growth risk of nontraded industries tends to have a stronger impact on domestic interest rate than its output growth risk of traded industries. Intuitively, this is because country-specific traded risk can be diversified in the global pool of traded goods before it affects prices in any country. The basic regression testing this relationship employs national output data at the industry level. We use binary dummies to classify the nontradability of the industries

$$r_{i,t}^H = \alpha + \beta_\sigma (\sigma_{i,t}^H)^2 + \beta_d d_{i,t} + \beta_{\sigma d} (\sigma_{i,t}^H)^2 d_{i,t} + \beta_x X_{i,t}^H + \epsilon_{i,t}^H,$$

[15] Inflation is computed as the year-to-year percentage change of the consumer price index, and the latter is sourced from OECD "Key Short-term Economics Indicators" database. Furthermore, inflation volatility is computed as standard deviation of the inflation growth. This control variable aims to address the fact that the real asset pricing model does not capture the possible effects from inflation risk.

where $r^H_{i,t} = r^H_t$ is country H's interest rate and thus independent of industry type i, $d_{i,t}$ is non-tradability dummy ($d_{i,t} = 1$ for nontraded industries and 0 otherwise, as we explain below). Table 4 reports the results associated with this regression. Countries' real annual industry-level outputs are constructed from the "OECD Structural Analysis (STAN)" database. An industry i is classified as nontraded ($d_{i,t} = 1$) if it belongs to one of the following ISIC 4 classes: 41-43 (construction); 45-56 (Wholesale and retail trade; Repair of motor vehicles and motorcycles; Transportation and storage; Accommodation and food service activities); 58-63 (Information and communication); 64-66 (Financial and insurance activities); 68-82 (Real estate, renting and business activities); 84-99 (Community, social and personal services). Other industries are taken as traded ($d_{i,t} = 0$). We divide the entire time period 1970-2016 into five decade-periods, and the volatility of per-capita output growth for each industry is computed as the respective standard deviation over each period. As before, the control variables include per-capita GDP mean growth, GDP size, and inflation volatility.

The key observation from Table 4 is that the slope coefficients associated with the interaction term (variance × dummy) are always negative.[16] When we take into account the GDP growth (to control for the intertemporal consumption smoothing effect), economy size, and inflation risk effects, these coefficients are statistically negatively significant. The negative sign of the interaction term coefficients fits the basic economic intuition that the nontraded output growth risk is more impactful to the economy than that of the traded output. Consequently, the output risk enhances the value of risk-free bonds, and depresses risk-free rate more aggressively when the risk comes from a nontraded industry.

3.2.3 Tests Using Multiple Industry Outputs and Global Nontradabilities

Some industries are not clear-cut traded or nontraded as depicted by a binary dummy of the above regression. In this section, we use continuous-valued global nontradability at industry level to account for this fine distinction. The hypothesis to be examined here is the same as above, namely all else being equal, output risk of nontraded industries matter more to country's interest rate than that of traded industries. The basic regression testing this relationship reads

$$r^H_{i,t} = \alpha + \beta_\sigma(\sigma^H_{i,t})^2 + \beta_\tau \tau_{i,t} + \beta_{\sigma\tau}(\sigma^H_{i,t})^2\tau_{i,t} + \beta_x X^H_{i,t} + \epsilon^H_{i,t},$$

[16] The first regression without any control variables gives a positive coefficient, but this coefficient is not statistically significant.

where τ_i is a global measure of nontradability of industry i. We adopt the standard definition of tradability as the ratio of OECD aggregate trade over OECD aggregate output of the industry i, and nontradability is the complement to tradability

$$\tau_i = 1 - \frac{\sum_{\text{OECD countries}} [i\text{'s import} + i\text{'s export}]}{2 \times \sum_{\text{OECD countries}} i\text{'s output}}. \tag{20}$$

Table 5 reports the results associated with this regression. Data sources are identical to those employed in the above regression. We use country-specific output series to compute country-specific industry i's growth volatility over each of five decade-periods. We aggregate these series to compute the global tradability and nontradability for each of good i.

The key observation from Table 5 is that the slope coefficients associated with the interaction term (variance $\times$ nontradability) are always negative. When we take into account the GDP growth (to control for the intertemporal consumption smoothing effect), economy size, and inflation risk effects, these coefficients are still statistically significant. The negative sign precisely fits the basic economic intuition that as countries mostly internalize their own nontraded shocks, the fluctuations in nontraded industries are more serious risk to the economy than those of the traded ones. Furthermore, output volatility act to lower risk-free rate. Consequently, risk-free rate is more sensitive (and negatively related) to output risk of industries of higher nontradabilities (defined at the OECD level).

3.2.4 Tests Using Multiple Industry Outputs and Country-specific Nontradabilities

In some situation, global measure of tradability does not exactly reflect the tradability of an industry at country level. This happens, e.g., when the trade levels are highly heterogeneous across countries in certain industries. To account for this fine distinction, in this section, we use continuous-valued country-specific nontradability at industry level. The hypothesis to be examined here is the same as above, namely all else being equal, output risk of nontraded industries matter more to country's interest rate than that of traded industries. The basic regression testing this relationship reads

$$r^H_{i,t} = \alpha + \beta_\sigma (\sigma^H_{i,t})^2 + \beta_\tau \tau^H_{i,t} + \beta_{\sigma\tau} (\sigma^H_{i,t})^2 \tau^H_{i,t} + \beta_x X^H_{i,t} + \epsilon^H_{i,t},$$

where τ^H_i is a country-specific measure of nontradability of industry i. We adopt the standard definition of tradability as one half of the ratio of national trade over national output of the industry i, and nontradability τ_i^H is the complement to tradability

$$\tau_i^H = 1 - \frac{[i\text{'s import} + i\text{'s export}]\ \text{by country } H}{[2 \times i\text{'s output}]\ \text{by country } H}. \tag{21}$$

Table 6 reports the results associated with this regression. Data sources are identical to those employed in the above regression. We use country-specific output series to compute both country-specific industry i's growth volatility over each of five decade-periods and i's country-specific tradability and nontradability.

The key observation from Table 6 is that the slope coefficients associated with the interaction term (variance × nontradability) are always negative. When we take into account the GDP growth, economy size, and inflation risk effects, these coefficients are still statistically significant. The negative sign precisely fits the basic economic intuition that as countries mostly internalize their own nontraded shocks, the fluctuations in nontraded industries are more serious risk to the economy than those of the traded ones. Furthermore, output volatility lowers risk-free rate via precautionary savings mechanism. Consequently, risk-free rate is more sensitive (and negatively related) to output risk of industries of higher nontradabilities (defined at the country level).

3.3 Carry Trade Returns

The evidences above shows that nontraded risk is an important factor behind national asset returns. Taking a step further, as *international* investment strategies are exposed to country-specific nontraded as well as aggregate traded risks, the strategies' risk premia should reflect the interplay of these risk factors, as well as their interaction with countries' sizes. This is because smaller countries find it easier to adjust their traded consumptions to manage their own nontraded output growth risks. By the same reason, nontraded output growth risks of larger countries exert stronger impacts on their trade partners and international markets. In this section, we investigate the empirical relationship between carry trade expected returns and nontraded output risk of the countries involved. Specifically, we devise two sets of tests which involve (i) evaluating currency portfolios based on one- and two-dimensional sorts along the variance of nontraded output growth, the economy's real size, and their product (Proposition 2) and (ii) constructing nontraded and traded consumption risk factors to price carry trades (Proposition 3). The valuation of all carry trades is exclusively from the perspective of U.S. investors, for whom the returns are in the denomination of U.S. dollars.[17]

[17] Maurer et al. (2019) give an account of the carry trade pricing in different currency denominations.

3.3.1 Currency Portfolios Based on the Nontraded Output Risk and Economy's Size

The theoretical analysis of Section 2.2 (Equation (13)) clearly indicates that controlling for all else, carry trades that borrow U.S. dollar and lend currencies of countries of less volatile nontraded outputs and smaller sizes tend to yield higher expected returns (and vice versa) to U.S. investors. To directly verify this structural relationship, stated in Hypothesis 3 above, we construct currency portfolios based on the variances of nontraded output growths and economic sizes as suggested by the theory. Forming portfolios helps to suppress the noises in individual currency returns, and delivers large and stable return spreads between portfolios by rebalancing (Lustig and Verdelhan (2007)).

We consider carry trade returns from U.S. investors' perspectives. For each country, we identify nontraded output as the outputs on services and construction. This output data is available at quarterly frequency, and sourced from OECD's "Quarterly National Accounts" (QNA) database.[18] For the one-dimensional sort, at the beginning of each quarter t, countries are sorted into four (quartile) portfolios based on the value of one of the following three characteristics:

(i) country-specific per-capita nontraded real output growth variance, or

(ii) relative real GDP size, defined as the ratio of a country's real GDP over the total real GDP of all countries in the sample, or

(iii) their product.

Portfolio 1 contains countries with lowest value of the characteristic being employed in the sorting, and portfolio 4 the highest. After portfolios' currency compositions are known at the beginning of quarter t, U.S. investors short U.S. dollars and long equally weighted portfolios P ($P \in \{1, 2, 3, 4\}$) of foreign currencies to earn the quarterly returns $XR_{t+1}^{-US,+P}$ realized at the beginning of quarter $t+1$. Following the literature, we assume investors carry out this strategy by entering uncovered long position in 3-month forward exchange contracts of foreign currencies F_i. We denote spot and 3-month forward exchanges rates as USD per unit of currency i at time t by $S_{i,t}$ and $X_{i,t}$. In the data, these rates are given in nominal terms. After factoring out inflation components, the 3-month

[18] To obtain a more extensive historical data, however, U.S. quarterly data are sourced from U.S. Bureau of Economic Analysis. See data appendix for further details.

realized bilateral carry trade real return between currency i and the USD (denominated in USD) is

$$XR_{t+1}^{-US,+F_i} \equiv \ln\left(\frac{S_{i,t+1}}{X_{i,t}}\right) = fd_{i,t} + \Delta s_{i,t+1} + infl_{t,t+1}^{US} - infl_{t,t+1}^{F_i}, \tag{22}$$

where $fd_{i,t} = \ln\left(\frac{S_{i,t}}{X_{i,t}}\right)$ (known at time t) is the forward discount, $\Delta s_{i,t+1} = \ln\left(\frac{S_{i,t+1}}{S_{i,t}}\right)$ the exchange rate growth, and $infl_{t,t+1}^{I}$ country I's inflation, realized during period $(t, t+1)$.[19] The quarterly real returns of the carry trade portfolio is therefore

$$XR_{t+1}^{-US,+P} = \sum_{F_i \in P}^{K_P} \frac{1}{K^P} XR_{t+1}^{-US,+F_i}. \tag{23}$$

The inflation is constructed as log return from each country's quarterly consumer price index (CPI) data (sourced from OECD "Key Short-term Economics Indicators" STEI database), whereas the spot and forward exchange rates data is from Thomson Reuters' Datastream database, which is available from 1976. Finally, our portfolios are rebalanced each quarter, and the annualized real carry trade returns for each portfolio are obtained by compounding the quarterly counterpart values. We note that because the OECD's QNA data, OECD's STEI data and the Datastream exchange rate data is unbalanced (data start at different times for different countries, see data appendix), not all OECD countries are available at the same time for the purpose of portfolio sorting.

Figure 6 plots the three separate sets of mean annualized returns and Sharpe ratios of four equally weighted and sorted carry trade portfolios. The carry trade strategies borrow USD and lend equally weighted currencies in each portfolio. Each set of results corresponds to one (of the three, listed above) characteristic employed in the sorting. Table 7 reports associated statistics. Notably, the figure shows a monotonically inverse relationship for the set (in green color) of portfolios sorted on the product of nontraded output growth risk and economy's size. Quantitatively, portfolio 1 earns a mean annual real excess return of 2.16% (Sharpe ratio of 23.45%), and portfolio 4 a return of -1.07% (Sharpe ratio of -13.25%) to U.S. investors. Thus a long-short portfolio strategy (long portfolio 1, short portfolio 4) earns mean annual real excess return of 3.23%, and Sharpe ratio of approximately 45%. This empirical inverse relationship is supported by the risk-based rationale that carry trades (borrowing home and lending foreign currencies) are hedges against the foreign

[19] Under the premise of the covered interest rate parity (CIP), the forward discount is equal to the interest rate differential $fd_{i,t} = \ln\left(\frac{R_{i,t}}{R_{US,t}}\right)$, where $R_{US,t}$ and $R_{i,t}$ are 3-month interest rate in the USD and currency F_i. In computing real carry trade return $XR_{t+1}^{-US,+F_i}$ to the U.S. investors, the inflation components in (22) undo nominal gains and losses due to the inflation realized in the U.S. and country F_i during period $(t, t+dt)$. Our results are robust to a replacement of the forward discount by the interest rate differential, i.e., to the CIP assumption.

nontraded output risks (coupled and amplified by the foreign economy's size), offering lower mean returns when the coupling is larger (Hypothesis 3). Currency portfolios sorted on sizes (in blue color) or nontraded output risk (in red color) alone do not exhibit a strictly monotonic pattern, consistent with the theoretical implication (13) that each of these characteristics on its own does not necessarily dominate the currency premia.

To control for sizes, Figure 7 plots mean annualized returns and Sharpe ratios of two-by-two equally weighted and sorted carry trade portfolios. The currencies are sorted into these portfolios along two characteristics (two-dimensional sort), namely, the variance of nontraded real output growth and relative real GDP size (defined above). The carry trade strategies borrow USD and lend equally weighted currencies in each portfolio. Table 8 reports associated statistics. Separately for currencies associated with either small or large size economies, the mean returns of portfolios decrease with the magnitude of nontraded real output growth risk. Similarly, separately for currencies associated with either low or high nontraded real output growth risk, the mean returns of portfolios decrease with size. These patterns provide the robustness for the risk-based rationale of currency premia (Hypothesis 3).

3.3.2 Linear Factor Analysis: Empirics

The theoretical analysis of Section 2.2 suggests another intuitive way to consider nontraded and traded consumption risks as two key pricing factors. From U.S. investors' perspectives, fluctuations in the U.S. nontraded output are a consumption risk and command a positive factor price (Proposition 3) as the nontraded output is consumed domestically. In contrast, the sign of the factor price associated with the movements in the U.S. traded consumption is an empirical matter because it depends on the interaction between economic sizes, country-specific nontraded outputs and global traded outputs in equilibrium. This section estimates these two factor prices empirically. We note that the payoffs of currency strategies that correlate with these consumptions are priced, resulting in carry trade risk premia. Accordingly, we employ sorted currency portfolios of the previous section as test assets in the estimation of the factor prices. We discuss, in order, the estimation procedure, data, and estimation results.

We empirically identify the U.S. traded and nontraded consumption variations as risk factors for U.S. investors; $f^{US}_{T,t+1} = \frac{C^{US}_{T,t+1} - C^{US}_{T,t}}{C^{US}_{T,t}}$, $f^{US}_{N,t+1} = \frac{C^{US}_{N,t+1} - C^{US}_{N,t}}{C^{US}_{N,t}}$. First, an application of the generalized method of moments (GMM) on Euler pricing equations for test asset (carry trade excess) returns $XR_{t+1}^{-US,+P}$ (23) helps to estimate the factor loadings $\{b_T, b_N\}$.[20] Then follow the factor prices $\{\lambda_T^{US}, \lambda_N^{US}\}$, and portfolios P's exposures $\{\beta_T^{US,P}, \beta_N^{US,P}\}$ to the U.S. traded and nontraded consumption growth risk (by replacing the single currency F in (16) by currency portfolio P)

$$\begin{bmatrix} \lambda_T^{US} \\ \lambda_N^{US} \end{bmatrix} = \left[Cov(\vec{f}^{US}, \vec{f}^{US})\right] \begin{bmatrix} b_T \\ b_N \end{bmatrix}; \quad \begin{bmatrix} \beta_T^{US,P} \\ \beta_N^{US,P} \end{bmatrix} = \left[Cov(\vec{f}^{US}, \vec{f}^{US})\right]^{-1} \begin{bmatrix} Cov(f_T^{US}, XR^{-US,+P}) \\ Cov(f_N^{US}, XR^{-US,+P}) \end{bmatrix},$$

Hence, the GMM procedure employed to estimate factor loading b's also estimates factor prices and risk exposures.

Currencies are sorted are sorted into four (quartile) portfolios based on the value of country-specific product of (real per-capita) nontraded output growth variance and relative real GDP size, as explained in the previous subsection. The risk factors f_T^{US}, f_N^{US} are computed as quarter-to-quarter percentage changes of the (real per-capita) U.S. traded and nontraded consumption respectively. The U.S. consumption and CPI series are from U.S. Bureau of Economic Analysis' "Quarterly U.S. consumption expenditures and price indexes". We identify the personal consumption expenditures on "services" as nontraded consumption, and on "goods" as traded consumption (see data appendix for further details).[21] After having constructed the quarterly series of portfolio returns $XR_t^{-US,+P}$ and factors $f_{T,t}^{US}$, $f_{N,t}^{US}$, we employ a two-stage GMM procedure to estimates factor loadings b_T, b_N jointly with the traded and nontraded factor prices λ_T^{US}, λ_N^{US}, and portfolio risk exposures $\beta_T^{US,P}$, $\beta_N^{US,P}$, as mentioned above and also in Menkhoff et al. (2012).[22] Their standard errors are determined from GMM-generated standard errors of factor loading b's and the delta method, as suggested by Burnside et al. (2011).

Table 9 reports the estimated factor prices. Both factor prices for traded and nontraded risk are positive and statistically significant. Quantitatively, one additional unit of a currency strategy's exposure to the U.S. nontraded consumption risk (i.e., β_N increases by one) boosts the expected excess return on the strategy by 29.2 basis points. The positive estimate presents an empirical support to the finding of Proposition 3 that fluctuations in the U.S. nontraded consumption growths are a priced risk in currency strategies from the U.S. investors' perspective. One additional unit

[20]These Euler pricing equations are $E_t\left[\frac{M_{t+1}^{US}}{M_t^{US}} XR_{t+1}^{-US,+P}\right]=0$, where $dm_{t+1}^{US} \equiv \log \frac{M_{t+1}^{US}}{M_t^{US}}$ is a function of the loadings $\{b_T, b_N\}$: $dm_{t+1}^{US} = b_T f_{T,t+1}^{US} + b_N f_{N,t+1}^{US}$ (15).

[21]We do not use U.S. output series (in the output approach to GDP) for the current factor analysis. This is because for traded components, due to trades, the U.S. traded output is not the same as the U.S. traded consumption. And in the theory being tested, it is the consumption risk that matters for the pricing.

[22]We also use lagged values of the carry trade portfolio returns as instruments.

of exposure to the U.S. traded consumption risk also boosts the expected excess return on the strategy by 26.8 basis points. This result shows that fluctuations in the U.S. traded consumption are also perceived as a risk by U.S. investors. The fact that estimates of λ_N^{US} and λ_T^{US} having similar order does not lessen the importance of nontraded output growth risks in asset pricing. This is because these risks contribute essentially to *both* factor prices (17), (18). Table 9 also reports the estimated consumption beta's for four currency portfolios. Values of betas vary across portfolios as well as factors, detailing the exposures of different currency strategies to two U.S. consumption risk factors. The exposures $\{\beta_N^P\}$ to the nontraded U.S. output growth risk do not necessarily sort along portfolios $\{P\}$ because the nontraded growth risk enters both traded and nontraded factor prices in equilibrium, per the discussion below (18). In a reduced-form approach, Lustig et al. (2011) document that two pricing factors feature prominently in FX markets. Among them, the slope factor, denoted HML_{FX} therein, is key to the pricing of currency risks. The nontraded risk factor is a priced factor in FX markets (Section 3.3), and explains the interest rate differential across economies (Section 3.2) which is the basis for the high-minus-low carry trade strategy.[23] The nontraded risk factor supports Lustig et al. (2011)'s finding and presents a possible structural interpretation for the slope factor in their reduced-form approach.

4 Conclusion

This paper examines the effects of national nontraded output growth risk on international asset pricing. Nontraded output growth risk is impactful because this output makes a large share of GDP and is consumed almost entirely by home population. In contrast, country-specific traded output growth risk can be diversified by means of commodity trades. Hence our analysis calls for a careful decomposition of GDP into traded and nontraded output components before assessing its role on the determination of asset prices.

Nontraded output shocks are nevertheless not entirely internalized by host countries because countries engage in international trades in other substitutable goods as well. While, to a certain extent, trades mitigate the impact of home nontraded output risk on the host economy, trades also transmit foreign nontraded output risks home, thus broaden the impact of nontraded output shocks among all trade partners in two-way directions. This mechanism contributes to the pricing

[23] The currency portfolios sorted on the nontraded risk overlap significantly with the currency carry trade portfolio sorted on the interest rate differential for this reason.

of all quantities in international markets, including interest rates, exchange rates, and currency premia. In contrast, as country-specific traded output risks are first pooled into the aggregate traded output, they are diversified fairly across countries, and largely drop from strategies involving off-setting positions in different national markets.

The frameworks in which a risk, apparently intrinsic to only one party, actually affects other parties are pervasive in the real world, e.g., social network settings and intermediary systems. The insights of nontraded risk would provide a pricing perspective to the emerging literature on financial networks. We hope to address these topics in future work.

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Table 1: Services' tradabilities, 1970-2016

Country	Measure	Construction (isic4 F)	Trade; repairs; transp., accom.; food serv. activ. (isic4 G-I)	Infor., commun.; financial; business activ. (isic4 J-N)	Community, Social, Personal serv. (isic4 O-U)
Australia	Tradability (%)	n.a.	0.00	0.01	0.12
	Fraction of GDP (%)	5.56	16.71	16.90	14.94
Czech Republic	Tradability (%)	4.01	15.73	0.63	0.11
	Fraction of GDP (%)	6.80	20.38	19.23	14.14
Denmark	Tradability (%)	2.79	13.05	0.71	0.06
	Fraction of GDP (%)	5.37	16.18	22.21	23.76
Euro	Tradability (%)	0.38	9.51	1.36	0.00
	Fraction of GDP (%)	4.30	16.31	26.04	17.87
Japan	Tradability (%)	n.a.	0.00	0.01	0.03
	Fraction of GDP (%)	5.66	21.02	19.93	14.08
Korea	Tradability (%)	10.47	7.97	0.19	0.07
	Fraction of GDP (%)	7.23	16.44	18.46	17.79
Luxembourg	Tradability (%)	17.18	15.89	0.32	0.09
	Fraction of GDP (%)	5.16	15.08	46.29	13.57
Norway	Tradability (%)	3.14	3.38	0.22	0.03
	Fraction of GDP (%)	4.09	15.47	16.74	19.72
Poland	Tradability (%)	2.17	12.77	0.90	0.06
	Fraction of GDP (%)	6.58	22.11	19.67	14.60
Slovakia	Tradability (%)	4.98	6.93	0.76	0.02
	Fraction of GDP (%)	6.36	19.73	17.55	13.99
Sweden	Tradability (%)	2.95	16.09	0.49	0.05
	Fraction of GDP (%)	4.55	15.23	23.99	19.83
United Kingdom	Tradability (%)	1.53	11.40	1.52	0.77
	Fraction of GDP (%)	7.00	16.56	25.78	17.42
United States	Tradability (%)	0.54	3.25	0.19	0.12
	Fraction of GDP (%)	4.88	14.90	33.28	15.23

Notes: This table lists the mean of country-specific tradabilities and sizes of different service categories for a representative set of 13 OECD countries, 1970-2016. Tradability of services is (one half of) the ratio of total real export and import over total real output of these services by a country (see (21)). Fraction of GDP (or size) of services is the ratio of total real output of these services over the real GDP of a country. See section 3.1 and data appendix for further details.

Table 2: Tradability of major U.S. industries, 1970-2016

	ISIC rev. 4 designation	Industries	US-specific tradability (%)	OECD tradability (%)
1	D13T15	Textiles, wearing apparel, leather and related products	112.2	94.63
2	D46	Wholesale trade, except of motor vehicles and motorcycles	50.69	49.89
3	D26T33	Machinery and equipment; Furniture; other manufacturing	42.39	30.61
4	D05T09	Mining and quarrying	38.32	39.64
5	D24T25	Basic metals and fabricated metal products	27.55	41.11
6	D19T23	Chemical, rubber, plastics, fuel, non-metallic mineral	25.82	42.35
7	D10T12	Food products, beverages and tobacco	22.68	36.53
8	D16T18	Wood and paper products, and printing	17.68	25.13
9	D01T03	Agriculture, hunting, forestry and fishing	17.36	16.91
10	D49T52	Transportation	6.36	2.87
11	D77	Rental and leasing activities	1.59	0.9
12	D64T66	Financial and insurance activities	1.01	0.62
13	D55T56	Accommodation and food service activities	0.78	0.4
14	D41T43	Construction	0.49	0.19
15	D85	Education	0.04	0.02

Notes: This table lists the tradabilities of major U.S. industries, along with their US-specific and OECD tradabilities. The industries are classified by ISIC Revision 4. US-specific tradability is (one half of) the ratio of total real export and import over total real output by the U.S. of the industry (see (21)). OECD tradability for a industry is defined similarly, but with export, import, and output replaced by total-OECD counterparts (see (20)). See section 3.1 and data appendix for further details.

Table 3: Trade-closedness regression

	(1)	(2)	(3)	(4)
Growth variance	-.10071	-.00575	.16469	.27006
	(3.5529)	(3.9556)	(3.8726)	(3.8588)
Closeness	-.0229**	-.02305**	-.02358**	-.02359**
	(.01069)	(.01012)	(.01018)	(.01022)
Variance × Closeness	-15.928	-16.331***	-16.645***	-16.825***
	(10.061)	(5.38)	(5.2726)	(5.3724)
Growth mean		-.05565	-.05648	-.03103
		(.74611)	(.74804)	(.74901)
GDP size			-.19958	-.18546
			(.13738)	(.14104)
Inflation volatility				.00043*
				(.00024)
Constant	.01605	.01772	.02066	.01945
	(.01385)	(.01276)	(.01309)	(.01319)
Adjusted R^2	0.35395	0.34146	0.32937	0.31859
N	93	93	93	93

Notes: Panel regression $r_t^H = \alpha + \beta_\sigma^H (\sigma_t^H)^2 + \beta_C^H \mathcal{C}_t^H + \beta_{\sigma C}^H (\sigma_t^H)^2 \mathcal{C}_t^H + \beta_x X_t + \epsilon_t^H$ to examine the effects of output volatility σ^H and trade closedness $\mathcal{C}^H$ on interest rate r^H. Panel regression includes country and time fixed effect. Values in parentheses below each regression coefficient are robust standard errors. 10%, 5%, 1% significance levels of two sided t-statistic are indicated by *, ** and ***, respectively.

Dependent variable is the annualized real interest rate, proxied by the short-term Treasury bill rate minus inflation rate, averaged over each period. The sample consists of annual data series for OECD countries 1970-2016. Current members of European Monetary Union are dropped from the sample when they joined the Union, and replaced by a single observation for Eurozone. Growth variance is the annualized variance of growth rate of per-capita real GDP. Closedness is one subtracted by the ratio of country's total real trade over country's real GDP (see (19)). Growth mean is the annualized mean of growth rate of per-capita real GDP. GDP size is the ratio of country's real GDP over total real GDP of OECD member states. Inflation volatility is the standard deviation of country's inflation growth. All mean and variance are computed for each of the non-overlapping periods 1970-1979, 1980-1989, 1990-1999, 2000-2009, 2010-2016. See data appendix for further details.

Table 4: Multi-industry nontradability-dummy regression

	(1)	(2)	(3)	(4)
Growth variance	.0088	.01581**	.01782***	.01884***
	(.00793)	(.00775)	(.00631)	(.00517)
Nontradability dummy	-.01436	-.00881	-.01001	-.01174
	(.00993)	(.01006)	(.00977)	(.00996)
Variance × dummy	4.7e-06	-.00798***	-.00838***	-.00734**
	(.00445)	(.00198)	(.0026)	(.0035)
Growth mean		.01923***	.01784***	.01495**
		(.00683)	(.00673)	(.00606)
GDP size			-.32274	-.34729
			(.35161)	(.33836)
Inflation volatility				-.00092**
				(.00045)
Constant	.03504***	.03492***	.03864***	.03924***
	(.00831)	(.00828)	(.00733)	(.00731)
Adjusted R^2	0.52077	0.52209	0.52781	0.54151
N	1475	1475	1404	1404

Notes: Panel regression $r^H_{i,t} = \alpha + \beta_\sigma(\sigma^H_{i,t})^2 + \beta_d d_{i,t} + \beta_{\sigma d}(\sigma^H_{i,t})^2 d_{i,t} + \beta_x X^H_{i,t} + \epsilon^H_{i,t}$ to examine the effects of industry-level output volatility σ^H_i and its dummy nontradability d_i on interest rate r^H. Panel regression includes country and industry fixed effect. Values in parentheses below each regression coefficient are robust standard errors. 10%, 5%, 1% significance levels of two sided t-statistic are indicated by *, ** and ***, respectively.

Dependent variable is the annualized real interest rate, proxied by the short-term Treasury bill rate minus inflation rate, averaged over each period. The sample consists of annual data series for OECD countries 1970-2016. Current members of European Monetary Union are dropped from the sample when they joined the Union, and replaced by a single observation for Eurozone. Growth variance is the annualized variance of growth rate of per-capita country-specific industries' real output. Nontradability dummies are at industry level; they assume value 1 for industries classified as nontraded sectors (Construction; Wholesale and retail trade; Repair of motor vehicles and motorcycles; Transportation and storage; Accommodation and food service activities; Information and communication; Financial and insurance activities; Real estate, renting and business activities; Community, social and personal services), and 0 otherwise. Growth mean is the annualized mean of growth rate of per-capita country-specific industries' real output over the corresponding period. GDP size is the ratio of country's real GDP over total real GDP of OECD member states. Inflation volatility is the standard deviation of country's inflation growth. All mean and variance are computed for each of the non-overlapping periods 1970-1979, 1980-1989, 1990-1999, 2000-2009, 2010-2016. See data appendix for further details.

Table 5: Multi-industry global nontradability regression

	(1)	(2)	(3)	(4)
Growth variance	.06069	.06354**	.06178**	.06037**
	(.04072)	(.02689)	(.02275)	(.02279)
Global nontradability	.00142	.00136	.00171	.00182
	(.002)	(.00211)	(.00209)	(.0021)
Variance × nontradability	-.05143	-.05457**	-.04947**	-.04851**
	(.03973)	(.02547)	(.02158)	(.02172)
Growth mean		.00219	-.00373	-.00439
		(.01375)	(.01274)	(.01276)
GDP size			-.47794*	-.4621*
			(.25782)	(.24861)
Inflation volatility				.00036
				(.00021)
Constant	.01098	.01097	.03733*	.03602*
	(.00782)	(.00783)	(.01897)	(.0188)
Adjusted R^2	0.60131	0.60092	0.60974	0.61546
N	944	944	909	909

Notes: Panel regression $r^H_{i,t} = \alpha + \beta_\sigma(\sigma^H_{i,t})^2 + \beta_\tau \tau_{i,t} + \beta_{\sigma\tau}(\sigma^H_{i,t})^2 \tau_{i,t} + \beta_x X^H_{i,t} + \epsilon^H_{i,t}$ to examine the effects of industry-level output volatility σ^H_i and its global nontradability τ_i on interest rate r^H. Panel regression includes industry as well as time fixed effect. Values in parentheses below each regression coefficient are robust standard errors. 10%, 5%, 1% significance levels of two sided t-statistic are indicated by *, ** and ***, respectively.

Dependent variable is the annualized real interest rate, proxied by the short-term Treasury bill rate minus inflation rate, averaged over each period. The sample consists of annual data series for OECD countries 1970-2016. Current members of European Monetary Union are dropped from the sample when they joined the Union, and replaced by a single observation for Eurozone. Growth variance is the annualized variance of growth rate of per-capita country-specific industries' real output. Here nontradability is a global measure and at industry level; it is one subtracted by one half of the ratio of global total real trade (i.e., import plus export) in an industry over the global total real output in that industry (see (20)). Growth mean is the annualized mean of growth rate of per-capita country-specific industries' real output. GDP size is the ratio of country's real GDP over total real GDP of OECD member states. Inflation volatility is the standard deviation of country's inflation growth. All mean and variance are computed for each of the non-overlapping periods 1970-1979, 1980-1989, 1990-1999, 2000-2009, 2010-2016. See data appendix for further details.

Table 6: Multi-industry country-specific nontradability regression

	(1)	(2)	(3)	(4)
Growth variance	.03111	-.02553	-.02175	-.02229
	(.03801)	(.03841)	(.03651)	(.0368)
Industry nontradability	-2.4e-05***	-2.4e-05***	-2.4e-05***	-2.4e-05***
	(3.7e-06)	(3.8e-06)	(3.9e-06)	(3.9e-06)
Variance $\times$ nontradability	-.00013***	-9.4e-05***	-9.3e-05***	-9.3e-05***
	(2.5e-05)	(2.9e-05)	(2.7e-05)	(2.7e-05)
Growth mean		-.05134	-.05272	-.05296
		(.0379)	(.03634)	(.03641)
GDP size			-.32268	-.30604
			(.27301)	(.25927)
Inflation volatility				.0002
				(.00054)
Constant	.00658	.00854	.02732	.02601
	(.0055)	(.0057)	(.01976)	(.01913)
Adjusted R^2	0.62277	0.62757	0.63225	0.63223
N	585	585	556	556

Notes: Panel regression $r_{i,t}^H = \alpha + \beta_\sigma (\sigma_{i,t}^H)^2 + \beta_\tau \tau_{i,t}^H + \beta_{\sigma\tau} (\sigma_{i,t}^H)^2 \tau_{i,t}^H + \beta_x X_{i,t}^H + \epsilon_{i,t}^H$ to examine the effects of industry-level output volatility σ_i^H and its country-specific nontradability τ_i^H on interest rate r^H. Panel regression includes country, industry as well as time fixed effect. Values in parentheses below each regression coefficient are robust standard errors. 10%, 5%, 1% significance levels of two sided t-statistic are indicated by *, ** and ***, respectively.

Dependent variable is the annualized real interest rate, proxied by the short-term Treasury bill rate minus inflation rate, averaged over each period. The sample consists of annual data series for OECD countries 1970-2016. Current members of European Monetary Union are dropped from the sample when they joined the Union, and replaced by a single observation for Eurozone. Growth variance is the annualized variance of growth rate of per-capita country-specific industries' real output. Nontradability is a country-specific measure and at industry level; it is one subtracted by one half of the ratio of country's real trade (i.e., import plus export) in an industry over the country's real output in that industry (see (21)). Growth mean is the annualized mean of growth rate of per-capita country-specific industries' real output. GDP size is the ratio of country's real GDP over total real GDP of OECD member states. Inflation volatility is the standard deviation of country's inflation growth. All mean and variance are computed for each of the non-overlapping periods 1970-1979, 1980-1989, 1990-1999, 2000-2009, 2010-2016. See data appendix for further details.

Table 7: Currency portfolios (one-dimensional sorting)

Sorted by NT x Size					
	Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4	Long-short (1)-(4)
Return (%)	2.159	0.685	-0.520	-1.067	3.226
Sharpe ratio (%)	23.450	7.257	-5.466	-13.253	44.763
Sorted by NT					
	Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4	Long-short (1)-(4)
Return (%)	1.243	0.680	1.305	-0.014	2.647
Sharpe ratio (%)	13.651	9.357	13.611	-14.571	36.492
Sorted by Size					
	Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4	Long-short (1)-(4)
Return (%)	1.030	0.276	-2.657	0.418	0.611
Sharpe ratio (%)	11.056	2.6448	-22.143	6.358	8.997

This figure presents (annualized) means and Sharpe ratios of real excess returns on four sorted quarterly rebalanced currency portfolios. The sample consists of quarterly data series for period 1976-2016. The portfolio are constructed by sorting currencies into four groups at beginning of quarter t based on the value of (1) nontraded real output growth variance $\times$ real GDP's relative size, (2) nontraded real output growth variance, and (3) real GDP's relative size, over the previous 15 quarters. Portfolio 1 contains currencies with the lowest sorted value, portfolio 4 the highest. Due to unbalances in spot and forward exchange rate series, countries' data become available at different times, and number of countries changes over time. See data appendix for further details.

Table 8: Currency portfolios (two-dimensional sorting)

Panel A. Return (%)			
	Small Size	Large Size	Long-short based on Size
Low NT	1.097	1.056	0.041
High NT	0.780	-3.036	3.816
Long-short based on NT	0.318	4.093	

Panel B. Sharpe ratio (%)			
	Small Size	Large Size	Long-short based on Size
Low NT	12.067	15.498	0.651
High NT	8.270	-24.575	32.667
Long-short based on NT	4.205	37.247	

This figure presents (annualized) means and Sharpe ratios of real excess returns on four quarterly rebalanced currency portfolios, as well as the resulting long-short portfolios to U.S. investors. The sample consists of quarterly data series for period 1976-2016. The sorted portfolios are constructed by sorting currencies into four groups at beginning of quarter t based on 2 dimensions, the real GDP's relative size and the value of nontraded real output growth variance over the previous 15 quarters. The long-short portfolios are constructed by either (i) keeping the same level (small/large) of real GDP relative size and long-short portfolios based on the value of nontraded real output growth variance, or (ii) keeping the same level (high/low) of nontraded growth variance and long-short portfolios based on GDP relative size. Due to unbalances in spot and forward exchange rate series, countries' data become available at different times, and number of countries changes over time. See data appendix for further details.

Table 9: Estimation of factor prices in linear factor models

		Nontraded consumption	Traded consumption
Factor prices (%)		0.292^{***} (0.0035)	0.268^{**} (0.132)
beta's	port. 1	-0.716	0.266
	port. 2	0.405	-0.117
	port. 3	-1.287	0.200
	port. 4	0.543	-0.297

Note: Upper panel reports the GMM annualized estimates of the factor prices (in percentage points), lower panel reports the estimates of the portfolios' exposures to risk factors (i.e. beta's) in the carry trade linear factor model using four quarterly rebalanced currency portfolios as test assets. HAC standard errors for the factor prices are obtained by two-stage GMM procedure using constant and lagged carry trade portfolio returns as instruments, and are reported in parenthesis. The currencies are sorted based on the product of nontraded real output growth variance and real GDP's relative size. The sample consists of quarterly data series for the period 1976-2016.

Figure 1: Nontraded real output-over-GDP ratio, averaged over 1970-2016, for OECD countries

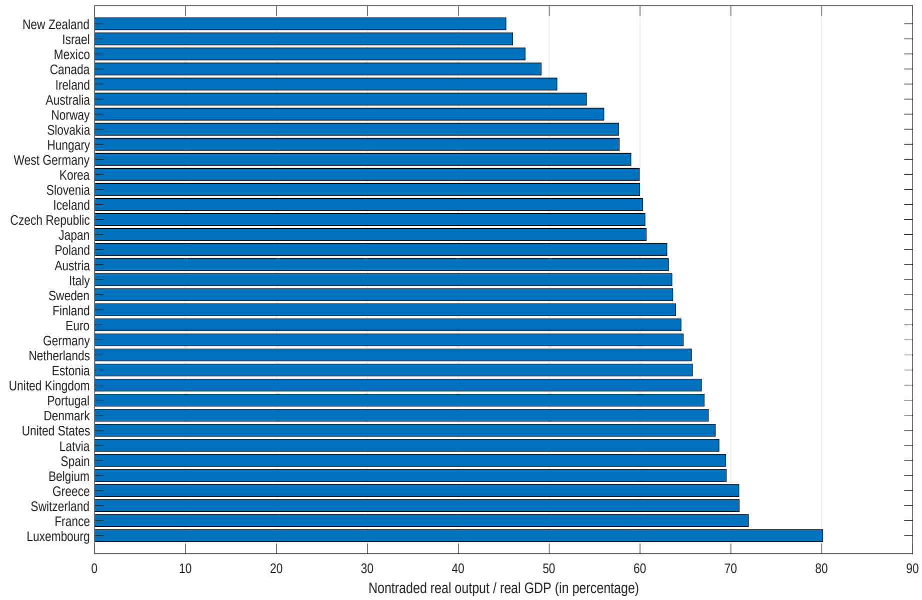


Figure 2: Nontraded output risk versus Size, averaged over 1970-2016, for OECD countries

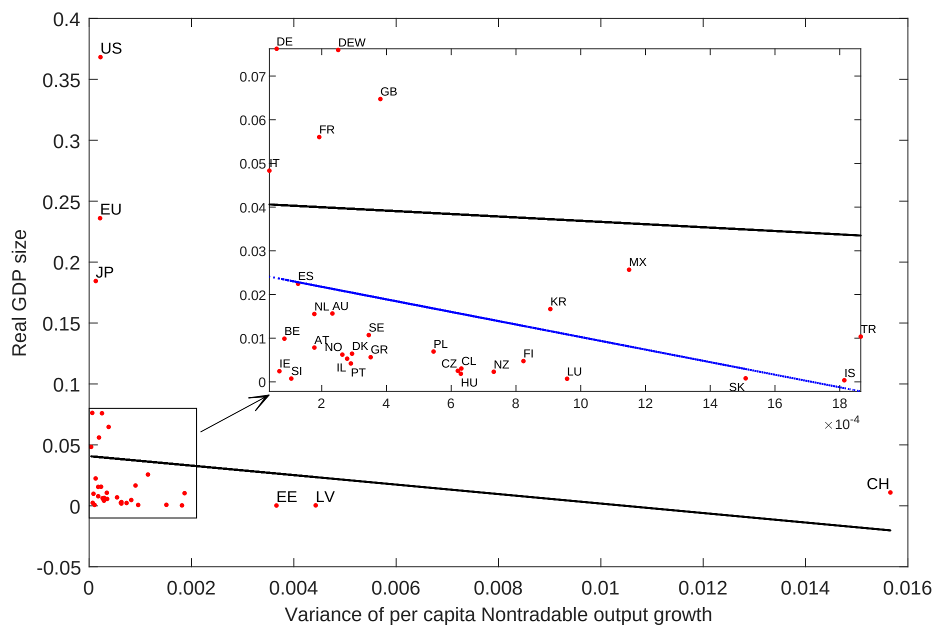


Figure 3: The combined effect of NT output risk and Size, averaged over 1970-2016, for OECD countries

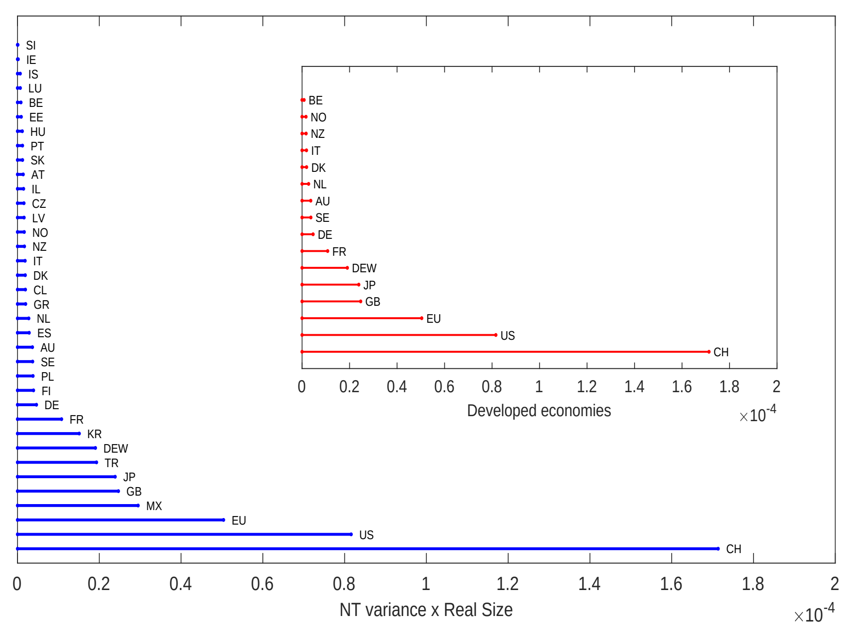


Figure 4: Volatility of per-capita nontraded real output growth, averaged over 1970-2016, for OECD countries

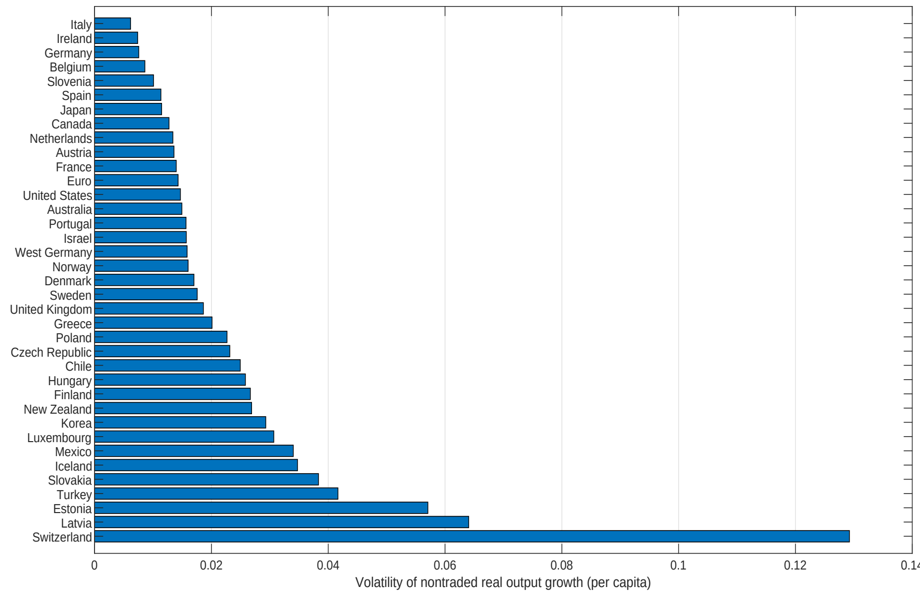


Figure 5: Real Trade-to-GDP ratio (i.e., openness), averaged over 1970-2016, for OECD countries. Trade is defined as the sum of export and import of the country.

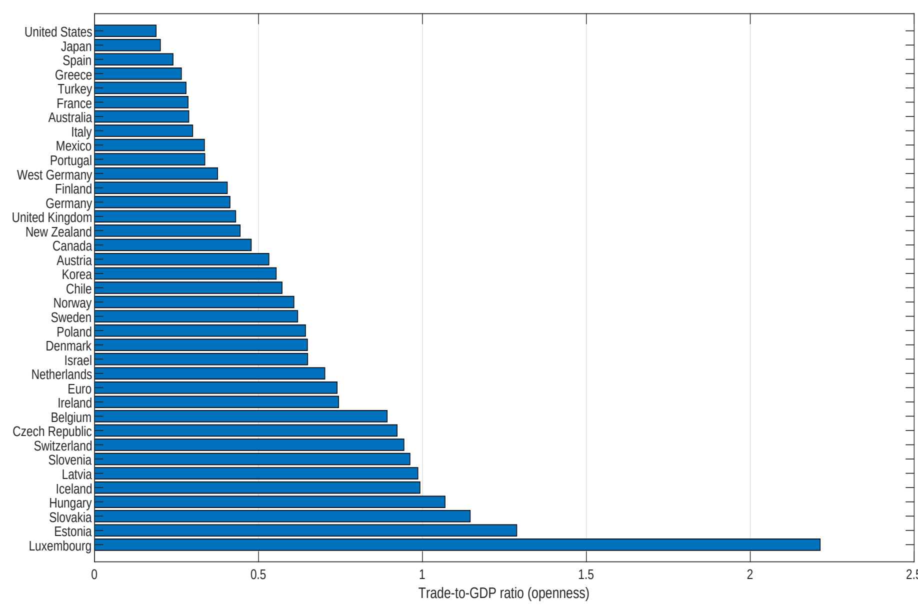
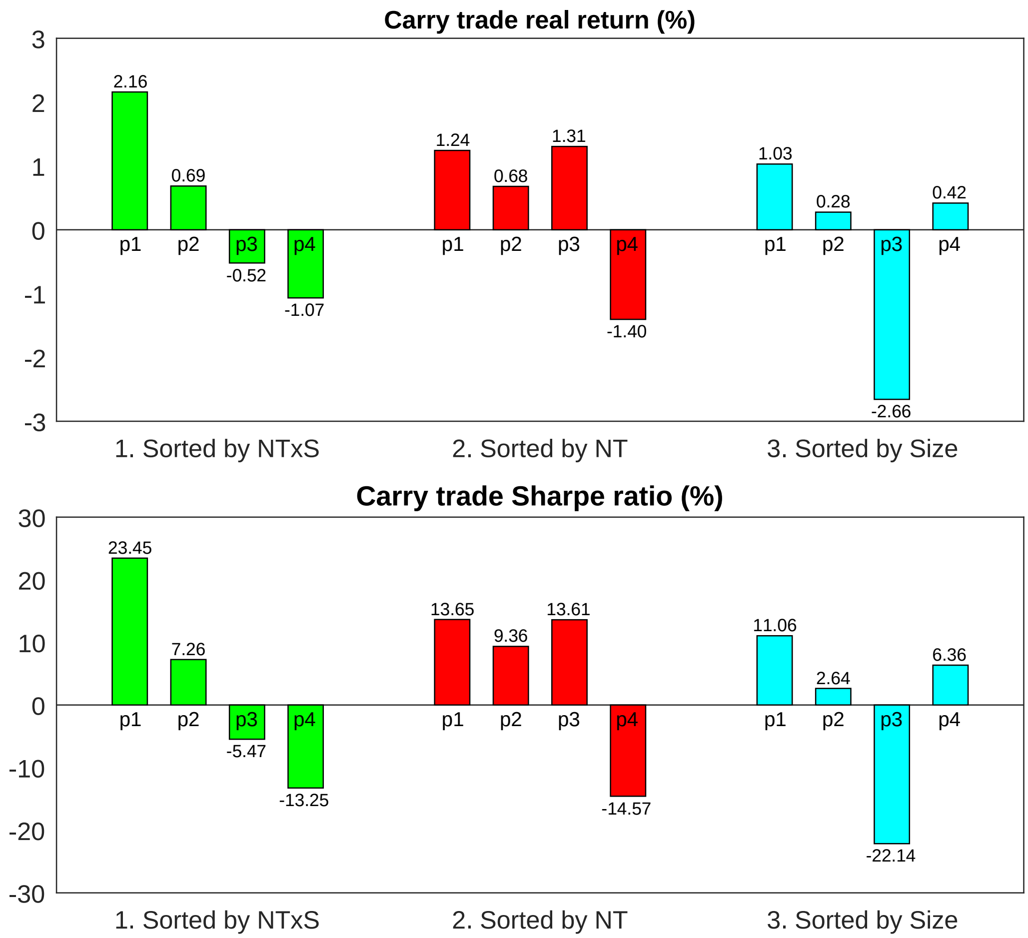
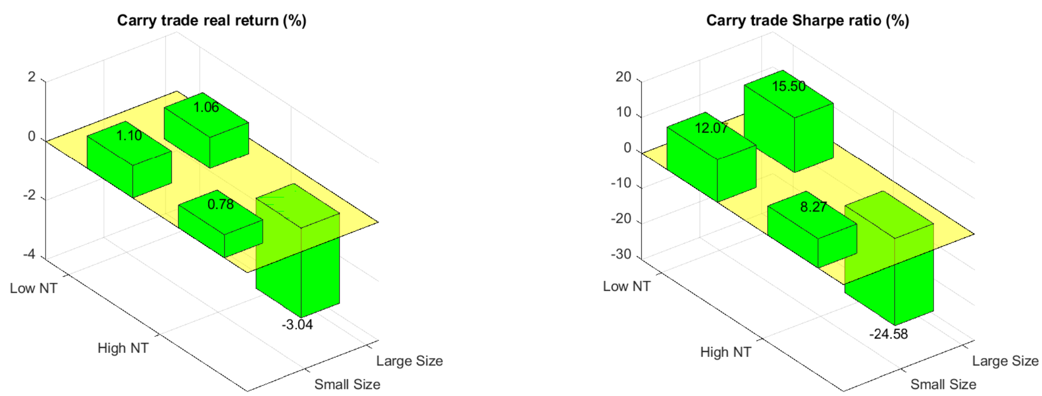


Figure 6: Carry trade real excess returns and Sharpe ratios for portfolios sorted on nontraded output risk and size



This figure presents (annualized) means and Sharpe ratios of real excess returns on four quarterly rebalanced currency portfolios to U.S. investors. The sample consists of quarterly data series for period 1976-2016. The portfolio are constructed by sorting currencies into four groups at beginning of quarter t based on (1) the value of nontraded real output growth variance $\times$ real GDP's size (2) the value of nontraded real output growth variance, and (3) real GDP's size. Portfolio 1 contains currencies with the lowest sorted value, portfolio 4 the highest. Due to unbalances in spot and forward exchange rate series, countries' data become available at different times, and number of countries changes over time. See data appendix for further details.

Figure 7: Two-dimensional Sort



This figure presents (annualized) means and Sharpe ratios of real excess returns on four quarterly rebalanced currency portfolios to U.S. investors. The sample consists of quarterly data series for period 1976-2016. The portfolios are constructed by sorting currencies into four groups at beginning of quarter t based on 2 dimensions, the real GDP's size and the value of nontraded real output growth variance. Due to unbalances in spot and forward exchange rate series, countries' data become available at different times, and number of countries changes over time. See data appendix for further details.

Appendices

A Data Sources

The empirical part of the current paper concerns countries that belong to the Organisation for Economic Co-operation and Development (OECD) because we reasonably expect that data quality for these developed economies should be higher than the rest of the world.

OECD countries: currently, there are 36 OECD member states listed in the following table (along with their country code).

Country	Code	Country	Code	Country	Code
Australia	AU	Greece	GR	New Zealand	NZ
Austria	AT	Hungary	HU	Norway	NO
Belgium	BE	Iceland	IS	Poland	PL
Canada	CA	Ireland	IE	Portugal	PT
Chile	CL	Israel	IL	Slovakia	SK
Czech Republic	CZ	Italy	IT	Slovenia	SI
Denmark	DK	Japan	JP	Spain	ES
Estonia	EE	Korea	KR	Sweden	SE
Euro	EU	Latvia	LV	Switzerland	CH
Finland	FI	Luxembourg	LU	Turkey	TR
France	FR	Mexico	MX	United Kingdom	GB
Germany	DE	Netherlandss	NL	United States	US
West Germany	DEW				

Eurozone countries: among OECD states, the following 15 belong to the Economic and Monetary Union (a.k.a., Eurozone or Euro area) with respective adopting date in the parenthesis; Austria (01/01/1999), Belgium (01/01/1999), Estonia (01/01/2011), Finland (01/01/1999), France (01/01/1999), Germany (01/01/1999), Greece (01/01/2001), Ireland (01/01/1999), Italy (01/01/1999), Luxembourg (01/01/1999), Netherlandss (01/01/1999), Portugal (01/01/1999), Slovak Republic (01/01/2009), Slovenia (01/01/2007), Spain (01/01/1999). (Only two other Eurozone states are Cyprus and Malta, but they do not belong to OECD and are not considered in the empirical analysis of the current paper.)

OECD "Aggregate National Accounts" (ANA) database, downloaded from stat.oecd.org, contains the following information used in our annual interest rate regression:

- "Gross domestic product" contains annual outputs as well as expenditures in national currency, and corresponding deflators. Data is available for the total gross domestic product (B1-GA), as well as for individual service categories, i.e., Construction (ISIC Rev.4 B1GVF); Distributional trade, repairs, transportation, accommodation, food service activities (B1GVG_I); Information and communication (B1GVJ); Financial and insurance activities (B1GVK); Real estate activities (B1GVL); Professional, scientific, technical activities, administrative, support service activities (B1GVM_N); Public administration, compulsory services, education, human health (B1GVO_Q); and Other service activities (B1GVR_U).
- "Export and import": export and import for the total economy, in local currency, as well as their deflators.
- "Population".
- "Exchange rate" data.

OECD "Quarterly National Accounts" (QNA) database, downloaded from stat.oecd.org, contains the following information used in our carry trade analysis:

- "Gross domestic product" contains quarterly outputs and expenditure in national currency, and corresponding deflators. The classification is the same as in the ANA database.
- Population: Turkey and Korea do not have quarterly data in the QNA, we use the annual data from ANA instead, and intrapolate for quarterly points. For other countries, not all countries have data starting from 1970. For those cases, similarly, we supplement by annual population data and intrapolate to fill in the missing points.

OECD "Key Short-term Economics Indicators" (Key STEI) database, downloaded from stat.oecd.org, provides data on consumer price index. This series is available for both annual and quarterly frequency. Inflation is then computed as the percentage change of the price index. Inflation volatility is calculated as the standard deviation of inflation growth.

"Treasury bill rates" is downloaded from Thomson Reuters Datastream. The rate is 3-month interest rates at quarterly frequency. Not every countries have interest rate available from 1970:

Austria series starts in Q3 1999, Chile in Q3 2011, Czech Republic in Q4 2002, Denmark in Q3 1988, Estonia in Q1 2011, Finland in Q1 1987, France in Q2 1989, Ireland in Q1 1984, Israel in Q1 1995, Italy in Q2 1988, Japan in Q2 1993, Korea in Q4 2004, Latvia in Q3 1994, Luxembourg in Q1 2003, Mexico and Netherlands in Q1 1978, New Zealand in Q2 1985, Norway in Q2 2003, Poland in Q3 1993, Portugal in Q2 1993, Slovakia in Q3 1994, Slovenia in Q1 1999, Spain in Q2 1988, Sweden in Q3 1989, Switzerland in Q2 1974, Turkey in Q3 2006, United Kingdom in Q2 1985, United States in Q1 1972.

"Spot and 3-month forward exchange rates" is downloaded from Datastream. These data are provided by Barclays Capital and WM/Reuters (WMR). In cases where data for one currency is available from both sources, the longer series is used. Exchange rates data quoted against the USD is available from 1983. Exchange rate data quoted against the Pound is available from 1976. We use the Pound/USD exchange rate to convert these exchange rates against Pound to exchange rates against USD for the earlier period.

"Quarterly U.S. consumption expenditures and price indexes" are series from U.S. Bureau of Economic Analysis. Table 2.3.5. therein contains "Personal Consumption Expenditures by Major Type of Product". Table 2.3.4. contains "Price Indexes for Personal Consumption Expenditures by Major Type of Product". We identify the personal consumption expenditures on services (i.e., the component "Services" listed in these tables) as the U.S. nontraded consumption. We identify the personal consumption expenditures on other goods (i.e., the component "Goods" listed in these tables) as the U.S. traded consumption. These quarterly series start well before 1970 (all our empirical studies in the current paper concern periods starting in 1970 or later).

OECD "Trade by Enterprise Characteristics" (TEC) database contains international annual trade in goods data broken down by different categories of enterprises. This data not only gives total import and export data, but also gives extra-EU trade data, facilitating the calculation of trade data for the EU zone. TEC data gives import and export for different types of goods (from ISIC Rev.4 code 1 to 99, as well as from code A to S). However, data is only available from 2007. We will combine with with other import and export data available in STAN3 database to have a full historical time series of trade data in various classifications of goods and services.

OECD "Structural Analysis" (STAN) database provides, for each OECD country, the annual nominal output series (in national currency) and the corresponding deflator series for various industries. The ISIC Rev.4 revision (STAN4 - November 2017 edition) has 30 countries, and uses

2010 base year for the deflators. The ISIC Rev.3 revision (STAN3 - 2011 edition) has 33 countries, and uses 2000 base year for the deflators. However, STAN3 also provides country-specific annual nominal import and export series (in national currency) and the corresponding deflator series. Therefore we combine STAN4, STAN3 and TEC to obtain a full data set of real outputs as well as real import/export for various industries.

B Combine STAN3, STAN4 and TEC Data

As outlined in the previous section, we extract output, import and export data at industry level from OECD "Structural Analysis" (STAN) database and OECD "Trade by Enterprise Characteristics" (TEC) database. The table below shows the data availability for each dataset.

Dataset	ISIC classification	Data	Coverage and other notes
STAN3, ed. 2011	Rev. 3	Output, import, export, output deflator	33 countries, 1970-2009, base year: 2000
STAN4, ed. 2017	Rev. 4	Output, output deflator	30 countries, 1970-2016 base year: 2010
TEC	Rev. 4	Import, export	30 countries, 2007-2016

Our first step is to find a matching between ISIC (International Standard Industrial Classification) Rev.3 and Rev.4 classifications. Information on ISIC, different classification revisions, as well as link tables between them, can be found in https://unstats.un.org/unsd/classifications/Family/Detail/27. However, the coding change is not an one-to-one mapping. The following table gives an example of how goods code D26 - Manufacture of computer, electronic and optical product - is mapped to codes 24, 25, 30 - 33, and 36 in ISIC3.

ISIC4	ISIC3	Manufacture of
2610	2429	semifinished dice or wafers, semiconductors
2610	2520	technical laminates
2610	3000	computer modems
2610	3110	inductors
2610	3120	electronic connectors
2610	3130	cable-typed connectors
2610	3210	electronic components and boards
2610	3230	heads, phonograph needles
2620	3000	
2630	3190	burglar and fire alarm systems
2630	3220	communication equipment
2630	3230	infrared devices, pagers
2640	3230	consumer electronics
2640	3694	video game consoles
2651	3190	pulse generators
2651	3220	GPS equipment
2651	3312	measuring, testing, navigating and control equipment
2651	3313	industrial process control equipment
2652	3330	
2660	3311	
2670	3230	digital cameras
2670	3312	optical range finders
2670	3320	optical instruments and photographic equipment
2680	2429	

We therefore manually go through each individual ISIC codes to find the best matching. Table 10 shows the industries that we can have reasonably accurate matching that we use in our industry analysis.

The following rules are used in our matching.

- Convert STAN3 to STAN4 codes to combine databases. Aggregate codes according to Table 10 to ensure closed matching.

Table 10: Matching between ISIC Rev.3 and ISIC Rev.4

ISIC Rev.4 codes	ISIC Rev.4 industry names	Matching ISIC Rev.3 codes
D01T03	Agriculture, hunting, forestry and fishing	01-05
D05T09	Mining and quarrying	10-14
D10T12	Food products, beverages and tobacco	15-16
D13T15	Textiles, wearing apparel, leather and related products	17-19
D16T18	Wood and paper products, and printing	20; and 21-22
D19T23	Chemical, rubber, plastics, fuel products and other non-metallic mineral products	23-25; and 26
D24T25	Basic metals and fabricated metal products, except machinery and equipment	27-28
D26T28 + D29T30 + D31T33	Machinery and equipment; Transport equipment; Furniture; other manufacturing; repair and installation of machinery and equipment	29-33; and 34-35; and 36
D35+D36	Electricity, gas, steam and air conditioning supply Water collection, treatment and supply	40-41
D41T43	Construction	45
D46	Wholesale trade, except of motor vehicles and motorcycles	51
D49 + D50 + D51 + D52	Land transport and transport via pipelines; Water transport; Air transport; Warehousing and support activities for transportation	60-63
D53 + D61	Postal and courier activities; Telecommunications	64
D55T56	Accommodation and food service activities	55
D64T66	Financial and insurance activities	65-67
D77	Rental and leasing activities	71
D85	Education	80
D86T88	Human health and social work activities	85
D94	Activities of membership organizations	91
D96	Other personal service activities	93
D97T98	Activities of households as employers; undifferentiated activities of households for own use	95
D99	Activities of extraterritorial organizations and bodies	99

Note: This table gives the industry codes we use in our industry analysis, for those industries that we can match closely between ISIC Rev.4 and ISIC Rev.3 codes. For other codes, one code in Rev.4 matches to different components of various codes in Rev.3 of so no accurate matching can be done.

- For countries with no STAN4 information, use STAN3 data.
- For countries with both STAN4 and STAN3 information, use STAN4 data (only production data available). Match import and export data from STAN3 (data to 2007) and TEC (from 2008-2016).
- Manually go through deflator data for both STAN3 and STAN4. For any goods codes that do not have deflator data, use deflator data from a larger group-code to fill in. For example, Switzerland goods code 94 does not have deflator data, we use data for group-code 94-96 to fill in.
- Ensure we have the same deflator base year, either 2000 (used in STAN3) or 2010 (used in STAN4) when combined data.

C Costly Trade

In the model of iceberg transport cost (Samuelson (1954)), only a fraction of $\frac{1}{1+\theta}$ of the transported traded good arrives at the destination. Consider a single good traded among two countries $\{H, F\}$ of similar sizes. Parameter θ quantifies the tradability cost: a higher θ implies a higher trade cost, hence, a lower tradability of the good. The linearity in the iceberg transport costs is a key modeling advantage to keep market completeness intact. Consequently, the equilibrium is obtained by solving the static world optimization subject to appropriate global resource constraints

$$\begin{aligned} &\max_{\{C_H^H, C_F^H, C_H^F, C_F^F\}} U^H(C^H) + U^F(C^F) \equiv e^{-\rho t} \left[\frac{(C_H^H + C_F^H)^{1-\gamma}}{1-\gamma} + \frac{(C_H^F + C_F^F)^{1-\gamma}}{1-\gamma} \right] \\ &\text{s.t.} \quad C_H^H + (1+\theta) C_H^F = \Delta^H; \quad C_H^F \geq 0; \quad C_F^F + (1+\theta) C_F^H = \Delta^F; \quad C_F^H \geq 0, \end{aligned}$$

where $C^H = \{C_H^H, C_F^H\}$ are home consumption components that originate from home and foreign outputs, respectively (the foreign counterpart is $C^F = \{C_H^F, C_F^F\}$). Thus, C_F^H is the import by H, which derives from the original amount $(1+\theta)C_F^H$ exported from F. Similarly, C_H^F is the import by F and derives from the original amount $(1+\theta)C_H^F$ from H. Countries desire to trade to share their output risks. However, the transport cost may limit the risk sharing. Intuitively, if the cost outweighs the benefit of risk sharing, countries opt not to trade and instead fully internalize their endowment shock; $C_H^F = C_F^H = 0$. To determine the conditions for commodity market no-trade, assume these conditions are currently not met and that trades take place. Because the shipping incurs a cost, the imported good is always more expensive than the locally endowed good, and countries always deplete their endowed resource before reaching out to the imported resource. In other words, conditional on trades taking place, there are two mutually exclusive alternatives:

$$\textbf{case 1}\text{: H imports, F exports,} \qquad C_H^H = \Delta^H; \;\; C_F^H > 0; \;\; C_H^F = 0; \;\; C_F^F < \Delta^F,$$

$$\textbf{case 2}\text{: H exports, F imports,} \qquad C_H^H < \Delta^H; \;\; C_F^H = 0; \;\; C_H^F > 0; \;\; C_F^F = \Delta^F.$$

By symmetry, it suffices to study case 1. As long as trades take place, regardless of their "iceberg-melting" friction, marginal utilities are equalized across countries ($\frac{\partial U^H}{\partial C^H} = (1+\theta)\frac{\partial U^F}{\partial C^F}$), as are the interest rates. Suppose that home country is an importer (case 1) and trades take place, the variation of social planner's Lagrangian with respect to non-binding consumptions $\frac{\partial}{\partial C_F^H}$, $\frac{\partial}{\partial C_F^F}$ produces FOC $\left(C_H^H + C_F^H\right)^{-\gamma} = (1+\theta)\left(C_H^F + C_F^F\right)^{-\gamma}$. Combining this with binding consumption $C_H^H = \Delta^H$, $C_F^H = 0$, and market clearing condition $C_F^F + (1+\theta)C_F^H = \Delta^F$ yields

$$C_F^H = \frac{\Delta^F - (1+\theta)^{\frac{1}{\gamma}}\Delta^H}{(1+\theta) + (1+\theta)^{\frac{1}{\gamma}}}; \qquad C_F^F = \frac{(1+\theta)^{\frac{1}{\gamma}}\left[\Delta^F + (1+\theta)\Delta^H\right]}{(1+\theta) + (1+\theta)^{\frac{1}{\gamma}}}. \tag{24}$$

From this we can also find home SDP $M^H = e^{-\rho t}\left(\Delta^H + C_F^H\right)^{-\gamma}$. The risk-free rate r^H is the opposite to expected growth rate of M^H; $r^H = -\frac{1}{dt}E_t\left[\frac{dM^H}{M^H}\right]$. Plugging equilibrium consumption solutions (24) into M^H, and an application of Ito lemma yields (assuming independent endowments Δ^H, Δ^F)

$$r^H = \rho + \gamma\frac{(1+\theta)\mu^H\Delta^H + \mu^F\Delta^F}{(1+\theta)\Delta^H + \Delta^F} - \frac{1}{2}\gamma(\gamma+1)\frac{(1+\theta)^2(\sigma^H)^2(\Delta^H)^2 + (\sigma^F)^2(\Delta^F)^2}{[(1+\theta)\Delta^H + \Delta^F]^2} \tag{25}$$

which is a more explicit version of (10).

Trades require net positive home import $C_F^H > 0$, and commodity market no-trade prevails otherwise. We analyze these two regimes in turn.

No-trade regime: Combining cases 1 and 2 yields the following no-trade condition for the commodity market:

$$\text{No-trade conditions:} \qquad (1+\theta)^{-1} < \left(\frac{\Delta^H}{\Delta^F}\right)^{\gamma} < (1+\theta).$$

Costly trade regime: In contrast with the no-trade regime, when friction is moderate and home and foreign outputs are sufficiently different, countries choose to trade to share output risk. Without loss of generality, we continue with case 1 above, in which home is the importing country (or $C_F^H > 0$). Conditional on this being the case, $(1+\theta)^{-1} > \left(\frac{\Delta^H}{\Delta^F}\right)^\gamma$, the home unambiguously curbs its imports when transaction cost increases (C_F^H (24) decreases in θ when $C_F^H > 0$). However, interestingly, the inverse holds for the exporting country F for all realistic values of transport cost and risk aversion. Contingent on trades taking place, the foreign country actually boosts its export $(1+\theta)C_F^H$ when θ increases to compensate for the increasing loss in the transportation.[24] This is because, when home investors are risk averse, their net import C_F^H decreases less than linearly with the transport cost.

Focusing on the precautionary savings effect on the interest rate, we examine only concerning terms (reported in (10)) of the interest rate expression (25). As the transport cost θ increases, interest rates become increasingly sensitive to home output shocks and decreasingly sensitive to foreign output shocks; $\frac{\partial^2 |r|}{\partial\theta\partial|(\sigma^H)^2|} > 0$, $\frac{\partial^2 |r|}{\partial\theta\partial|(\sigma^F)^2|} < 0$. These behaviors, when combined with the earlier findings that $\frac{\partial C_F^H}{\partial\theta} < 0$ and $\frac{\partial\left[(1+\theta)C_F^H\right]}{\partial\theta} > 0$, precisely support our key *thesis* that when shocks are of a more nontraded nature (i.e., θ increases), they matter more to the country's asset prices. From the importing country H's perspective, a surge in trade cost θ coincides with a reduction in trades as its imports C_F^H drop. At the same time, the impact of the country's own volatility σ^H on its interest rate r^H increases while the impact of foreign volatility σ^F on r^H decreases. Likewise, from the exporting country F's perspective, a surge in trade cost coincides with a boost in trades as its export $(1+\theta)C_H^F$ increases. At the same time, the impact of its own volatility σ^F on its interest rate r^F decreases, whereas the impact of partner's volatility σ^H on r^F increases. Overall, the variation in trade frictions implies a structural relationship between nontradability and domestic asset prices summarized in Hypothesis 2.

D Derivations and Proofs

This appendix presents technical derivations of all results in the main text, concerning the basic model. In this basic model with complete market and no trade friction (Section 2), the equilibrium marginal utilities of traded consumption equal across countries, which give K FOCs; $M_T = \frac{\partial U^H}{\partial C_T^H}$ $\forall H = 1, \ldots, K$. The market clearing condition for traded good presents another equation to solve for $K+1$ unknowns; $\{C_T^H\}_{H=1}^K$ and M_T. We log-linearize the system to obtain an approximative

[24] $\frac{\partial}{\partial\theta}\left[(1+\theta)C_F^H\right] = \frac{\gamma-1}{\gamma}(1+\theta)^{\frac{1-2\gamma}{\gamma}}\Delta^F - \frac{\gamma-1}{\gamma}(1+\theta)^{\frac{2-2\gamma}{\gamma}}\Delta^H - \frac{1}{\gamma}(1+\theta)^{\frac{1-\gamma}{\gamma}}\Delta^H - \frac{1}{\gamma}(1+\theta)^{\frac{2-2\gamma}{\gamma}}\Delta^H$. For all realistic values of γ and θ, the last two terms are negligible compared with the second term. Then, the trade condition $C_F^H > 0$ immediately implies that $\frac{\partial}{\partial\theta}\left[(1+\theta)C_F^H\right] > 0$.

solution in closed form.

Equilibrium log consumption (5): Plugging the expression (2) for U^H into the FOC (4), and log-linearizing this FOC around the steady state corresponding to the symmetric configuration $\left\{\delta_T^H = \delta_N^H;\ \delta_T^H/\Lambda^H = \delta_T^F/\Lambda^F\right\}$ yield an approximate equation[25]

$$\begin{aligned} m_T &\approx \lambda^H - \rho t + (\epsilon - \gamma)(\omega_T c_T^H + \omega_N \delta_N^H) - \epsilon c_T^H + \log \omega_T \\ &= \lambda^F - \rho t + (\epsilon - \gamma)(\omega_T c_T^F + \omega_N \delta_N^F) - \epsilon c_T^F + \log \omega_T. \end{aligned}$$

Similarly, log-linearizing the traded good market clearing equation yields (where $\lambda = \log \Lambda = \log \sum_H^K \Lambda^H$)

$$\sum_H^K \frac{\Lambda^H}{\Lambda} c_T^H = \delta_T + \sum_H^K \frac{\Lambda^H}{\Lambda} \lambda^H - \lambda. \tag{26}$$

Substituting c_T^H from the first equation above into the second equation gives m_T, and then c_T^H in (5).

Country-specific stochastic discount factor (6): In pricing country-specific financial assets, the appropriate measures are country-specific consumption baskets (i.e., national currencies in the current consumption-based setting). A country-specific consumption basket is the lowest-cost bundle of traded and nontraded consumption that delivers a unit of country's utility, given the consumption goods' prices $\{P_T^H \equiv 1, P_N^H\}$ (in term of traded goods). The basket's composition $\{C_T^H, C_N^H\}$ and value P^H solve $\min_{C_T^H, C_N^H} P^H \equiv C_T^H + C_N^H P_N^H$ subject to $\left[\omega_T (C_T^H)^{1-\epsilon} + \omega_N (C_N^H)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} = 1$. Then follows the value of consumption basket in term of traded good

$$P_t^H = \left[\omega_T^{\frac{1}{\epsilon}} + \omega_N^{\frac{1}{\epsilon}} (P_N^H)^{\frac{1-\epsilon}{-\epsilon}}\right]^{\frac{-\epsilon}{1-\epsilon}};$$

From this and M_T above follows the identity in equilibrium $M_t P_t^H = M_t^H$, where $M_t^H \equiv \frac{\partial U^H}{\partial C^H} = e^{-\rho t}(C^H)^{-\gamma}$ and C^H is the country-specific consumption aggregator.[26] The current price of the country-specific risk-free bond (that pays one unit of country-specific consumption basket at time s) is

$$B_{t,s}^H = \frac{1}{P_t^H} E_t \left[\frac{M_s}{M_t} P_s^H\right] = E_t \left[\frac{M_s^H}{M_t^H}\right].$$

It is this pricing equation that establishes the above M_t^H as the country-specific SDF of country

[25] Lower-case letters denote logarithms, e.g., $m \equiv \log M$, $\lambda \equiv \log \Lambda$, $c = \log C$, $\delta = \log \Delta$.

[26] In contrast with the country-specific M^H, M_T is the marginal utility with respect to traded good and is same for all countries in complete market settings.

H. That is, prices computed using this SDF are in unit of country-specific consumption basket. Note that $m^H = \log M^H = -\rho t - \gamma \log C^H$. Log-linearizing the consumption aggregator C^H and using log equilibrium traded consumption c_T^H in (5) yield country-specific log SDF (6).

Proof 1 (Proof of Proposition 1) *From (6) follow the partial derivatives* $\frac{\partial m^H}{\partial \delta_N^H} = -\gamma\omega_N \left[1 - \alpha(\gamma-\epsilon)\omega_T\left(1 - \frac{\Lambda^H}{\Lambda}\right)\right]$ *and* $\frac{\partial m^F}{\partial \delta_N^H} = -\gamma\omega_N \left[\alpha(\gamma-\epsilon)\omega_T \frac{\Lambda^H}{\Lambda}\right]$. *Evidently,* $\left|\frac{\partial m^H}{\partial \delta_N^H}\right| > \left|\frac{\partial m^F}{\partial \delta_N^H}\right|$ *when* $\gamma - \epsilon > 0$ *(Footnote 7).*

Proof 2 (Proof of Equation (11) and Proposition 2) *We start with the differential representation for SDF* M^H

$$\frac{dM^H}{M^H} = -r^H dt - \eta^H dZ^H; \quad m^H = \log m^H \Longrightarrow dm^H = -\left(r^H + \frac{1}{2}(\eta^H)^2\right) - \eta^H dZ^H.$$

where η^H *is the home market price of risk. Similar relations hold for* M^F *and* m^F. *Plugging these into the realized carry trade excess return* $XR_{t+dt}^{-H,+F}$ *(upper equation in (11)), applying Itô's lemma and taking the conditional expectation yield*

$$\begin{aligned}
&E_t\left[XR_{t+dt}^{-H,+F}\right] = E_t\left[\frac{1+\frac{dM^F}{M^F}}{1+\frac{dM^H}{M^H}}(1+r^F dt) - (1+r^H dt)\right] \\
&= E_t\left[\left(1 + dm^F + \frac{1}{2}(dm^F)^2\right)\left(1 - dm^H + \frac{1}{2}(dm^H)^2\right)(1+r^F dt) - (1+r^H dt)\right] \\
&= E_t\left[dm^F + \frac{1}{2}(dm^F)^2 - dm^H + \frac{1}{2}(dm^H)^2 - dm^H dm^F + r^F dt - r^H dt\right] \\
&= (\eta^H)^2 - \eta^H\eta^F = -Cov_t\left[dm^H, dm^F - dm^H\right],
\end{aligned}$$

which is (11). Next, combining (6) and (12) implies the key expression for expected carry trade excess return (13) of Proposition 2.

Proof 3 (Proof of Proposition 3) *We first develop (16) to obtain more explicit expressions for* λ_T *and* λ_N

$$\lambda_T^H = Var(f_T^H)b_T + Cov(f_T^H, f_N^H)b_N; \qquad \lambda_N^H = Cov(f_T^H, f_N^H)b_T + Var(f_N^H)b_N.$$

Plugging $\{b_T, b_N\}$ *and* $\{f_T^H, f_N^H\}$ *from (15) into above expressions yields (17) of Proposition 3 and (18) for factor prices associated with nontraded and traded consumption growth risk respectively.*

E Online Appendix: Beyond Benchmark Model

The key intuition, developed alongside the basic setting of international finance in the main text, is that the country-specific traded output risk should have a smaller impact on asset prices than the country-specific nontraded output risk because of the diversification in the traded good market. However, the basic model possesses several simplifications, including (i) homogeneous consumption taste for a single common traded good and (ii) complete financial markets worldwide. In this section, we relax these assumptions, verify, and thus strengthen the above intuition to a more realistic and robust economic setting.

E.1 Arbitrary Trade Configuration

Generalized setup: In the current general setting, there are l varieties of traded goods and K types of nontraded goods, and each of the latter is consumed by one respective country. A particular type h of traded goods can be consumed only by some K_h countries. Similarly, a particular country H trades and consumes only some l^H varieties of traded goods. These features aim to capture the realistic and vastly different trade configurations among countries, as well as the vastly different popularity of different traded goods. Moreover, countries can also have country-specific tastes for the traded goods ($\{\omega_h^H\}$) and nontraded good (ω_N^H) that they consume, subject to the conventional normalization $\omega_N^H + \sum_h^{l^H} \omega_h^H = 1$. We also assume that the financial market is complete because contingent claims on all outputs and countries' risk-free bonds are available investment instruments. Consequently, the world's static optimization problem can be used to study the equilibrium behaviors of consumption allocations and asset prices in this economy.

$$\max_{\{C_{h,T}^H\}} \sum_{H=1}^{K} \Lambda^H \frac{e^{-\rho t}}{1-\gamma} \left[\sum_h^{l^H} \omega_{h,T}^H (C_{h,T}^H)^{1-\epsilon} + \omega_N^H (\Delta_N^H)^{1-\epsilon} \right]^{\frac{1-\gamma}{1-\epsilon}} \quad \text{s.t.} \quad \sum_H^{K_h} C_{h,T}^H = \Delta_{h,T} \quad \forall h = 1, \ldots, l.$$

Although a country may have different tastes for different goods that they consume, the substitutability between any two varieties, either traded or nontraded, is characterized by the same elasticity coefficient ϵ. It is apparent from the market market clearing conditions that only the aggregate outputs for traded good varieties directly enter the dynamic of the economy. However, the associated output shocks will have different impacts on different countries, depending on their country-specific trade configurations. The current complex setting calls for a quantitative analysis to shed light on the role of these shocks on consumption allocations and prices.

Equilibrium allocations: Combining log-linearization and iteration techniques yield the equilibrium log consumption c_h^H of traded good h by country H

$$\begin{aligned} c_h^H &= \delta_{h,T} - (\gamma-\epsilon)\alpha^H \omega_N^H \delta_N^H + (\gamma-\epsilon) \sum_J^{K_h} \alpha^J \frac{\Lambda^J}{\Lambda_h} \left(\omega_N^J \delta_N^J + \sum_j^{l^J} \omega_{j,T}^J \delta_{j,T} \right) \\ &- (\gamma-\epsilon)\alpha^H \sum_i^{l^H} \omega_{i,T}^H \left[\delta_{i,T} + (\gamma-\epsilon) \sum_I^{K_i} \alpha^I \frac{\Lambda^I}{\Lambda_i} \left(\omega_N^I \delta_N^I + \sum_k^{k^I} \omega_{k,T}^I \delta_{k,T} \right) \right], \end{aligned} \tag{27}$$

where in the current general setting

$$\Lambda_i \equiv \sum_I^{K_i} \Lambda^I; \qquad \alpha^H \equiv \frac{1}{(1-\omega_N^H)\gamma + \omega_N^H \epsilon} > 0 \tag{28}$$

are the good-specific relative size of the aggregate economies (those that consume good i) and a country-specific measure of weighted elasticity of consumption substitution, respectively. It is plausible that in this entangled trade network, many outputs affect country H's consumption of good h. In leading orders of importance, these include h's global supply ($\delta_{h,T}$); H's nontraded output (δ_N^H); nontraded output (δ_N^J) and traded global supply ($\delta_{j,T}$) consumed by any other country $J \in K^h$ that also consumes h; global supply ($\delta_{i,T}$) of any other traded good $i \in l^H$ consumed by H; and finally, the nontraded output (δ_N^I) and global supply ($\delta_{k,T}$) of traded goods k consumed by any country $I \in K^i$ that also consumes i.

Similar to the simpler setting of Section 2, a country's traded consumption allocation c_h^H increases with the global supply $\delta_{h,T}$, decreases with the host's nontraded output δ_N^H, and increases with nontraded output δ_N^J of all trade partners J in good h. As country H also consumes other traded variates $\{\delta_{i,T}\}_{i \in l^H}$, H's consumption c_h^H in good h tends to negatively correlate with shocks $dZ_{i,T}|_{i \neq h}$ through the substitution effect between any two traded goods. Furthermore, because the consumptions of all trade partners $J \in K_h$ in good h are tuned to the nontraded δ_N^J and traded global supplies $\{\delta_{j,T}\}_{j \in l^J}$ that they consume, these shocks are also positively compounded into c_h^H, again through trade (market clearing) and substitution effects.

Most interestingly, even in the current general trade network setting, the international transmission of output shocks follows a simple and intuitive quantitative pattern in the leading orders. That is, the transmission process involving trades in a good i with a mediating country I warrants a dampening coefficient,[27]

$$(\gamma-\epsilon)\alpha^I\frac{\Lambda^I}{\Lambda_i}=\frac{\frac{1}{\epsilon}-\frac{1}{\gamma}}{\frac{1-\omega_N^I}{\epsilon}+\frac{\omega_N^I}{\gamma}}\frac{\Lambda^I}{\Lambda_i}.$$

Here, $\frac{\Lambda^I}{\Lambda_i}$ characterizes the relative power of mediating country I in setting the global price for traded good i (through FOC), and $\left(\frac{1}{\epsilon}-\frac{1}{\gamma}\right)/\left(\frac{1-\omega_N^I}{\epsilon}+\frac{\omega_N^I}{\gamma}\right)$ quantifies how readily shocks in one consumption sector affect the others in a country.[28] Next, we examine the stochastic discount factors (SDFs) to explore how investors price the risk associated with these output shocks in different countries.

Equilibrium pricing: As shocks affect consumption allocations, they also move equilibrium prices accordingly to clear the market. The country H's log SDF is

$$\begin{aligned} m^H &= -\rho t-\gamma\sum_h^{l^H}\omega_{h,T}^H\delta_{h,T}-\gamma\omega_N^H\left[\delta_N^H-\sum_h^{l^H}(\gamma-\epsilon)\alpha^H\omega_{h,T}^H\left(1-\frac{\Lambda^H}{\Lambda_h}\right)\delta_N^H\right] \\ &- \gamma\sum_h^{l^H}\omega_{h,T}^H\sum_{J\neq H}^{K_h}(\gamma-\epsilon)\alpha^J\frac{\Lambda^J}{\Lambda_h}\left(\omega_N^J\delta_N^J+\sum_j^{l^J}\omega_{j,T}^J\delta_{j,T}\right) \end{aligned} \tag{29}$$

Reassuringly, all shocks that affect country H's consumptions are also priced by this stochastic discount factor. In particular, all traded and nontraded consumption shocks of H and any of it trade partners are compounded in m^H. As in the simpler case of Section 2, up to taste coefficients, the traded shocks are fully internationalized (in the aggregate output $\delta_{h,T}$) and spread uniformly to all countries $I\in K_h$ that consume good h. As $\omega_{h,T}^H$ generally drops with the number l^H of varieties consumed by H,[29] the country-specific traded shock of a particular variety matters even less to its country of origin in the current setting of multiple traded goods. In contrast, nontraded shocks are internalized, but not fully. As the second term within the square brackets shows, country H can tunnel its own nontraded shock in δ_N^H through trades in all l^H channels in which H participates. The ability to mitigate this shock through a particular channel h clearly decreases with a country's relative size $\frac{\Lambda^H}{\Lambda_h}$ in the world trade market for good h. Under mild home bias condition, country-

[27]As γ is (substantially) larger than ϵ, mild home bias conditions assure that $(\gamma-\epsilon)\alpha^I\frac{\Lambda^I}{\Lambda_i}<1$.

[28]Section 2 asserts that the difference $\frac{1}{\epsilon}-\frac{1}{\gamma}$ characterizes how willing a country is to substitute traded and nontraded consumptions to smooth its aggregate consumption. When this difference is large and positive as in the data, countries are flexible to make this substitution. As a result, a shock from one consumption sector is readily transmitted to the other sector. In the current setting, each country has one nontraded and several traded sectors, but all have the same pairwise substitution elasticity of ϵ.

[29]This is a consequence of the normalization condition $\omega_N^H+\sum_h^{l^H}\omega_{h,T}^H=1$.

specific nontraded shocks still matter more to the country's pricing than do the traded counterparts. Finally, we also see that traded shocks (in $\delta_{j,T}$) affecting any trade partner J are also factored in m^H. When H does not consume these goods, $j \notin l^H$, their shocks to H are similar the purely nontraded shocks of partners J.

E.2 Incomplete Market

In equilibrium, the complete financial markets equalize all countries' marginal utilities of the traded consumption and thus enable the optimal international risk sharing and consumption allocation. In reality, however, the financial markets of some countries are more developed than those of others, which should better facilitate these developed countries to manage their own as well as trade partners' output risk. Stylistically, because of either information asymmetry or lack of proper managerial enforcement, the equities associated with nontraded sectors of emerging economies are less marketable worldwide. It is interesting to explore the new qualitative implications of market incompleteness on international risk sharing and contrast them with those of the simplified complete market paradigm. Sarkissian (2003) is an earlier work that investigates imperfect consumption risk sharing and its effects on carry trade premia. To this end, we now analyze a stylized model in which nontraded output risk is the central factor behind the incompleteness in the financial markets.

Setup: We consider the world economy with perfect trades but an incomplete financial market. In the commodity sector, there are country-specific nontraded goods (one per country) and a single traded good (common to all countries). The traded good can be shipped globally without the friction, and thus only its aggregate output influences the pricing. Accordingly, we assume that the financial assets associated with the traded good sector are perfectly structured. That is, a stock S_T contingent on the aggregate output and a risk-free bond B_T paying one unit of traded good in the next period are available to investors worldwide. In contrast, the financial assets associated with nontraded sectors are incomplete. We assume that countries belong to either the "developed" or the "emerging" group. For any developed economy ($H \in \mathcal{D}$), the stock S_N^H contingent on the H's nontraded output and risk-free bond B_N^H paying one unit of H's nontraded good in the following period are also available to all investors. However, assets associated with nontraded sectors of emerging economies ($H \notin \mathcal{D}$) are not marketable and thus simply do not exist. In this framework, the world financial market is incomplete because there are more shocks than the available financial hedging instruments. To simplify the exposition, we assume a homogeneous size for all economies embedded in a two-period setting $\{t, t+1\}$, but maintain the heterogeneous consumption tastes $\{\omega_T^H, \omega_N^H\}_H$ across countries. Relaxing all of these assumptions is tedious but straightforward.

The most convenient choice for the numeraire in this setting is the traded good, which we adopt hereafter. Thus, in every period, all prices are in (contemporaneous term of) the traded good. Because the market is incomplete, we consider the optimization problem for each country.[30] Let $x_T^{HS}, x_T^{HB}, x_N^{HFS}, x_N^{HFB}$ denote the holdings of H's investor, respectively, in world stock S_T, world bond B_T, F's stock S_N^F, and F's bond B_N^F.

$$\max_{C_{T,t}^H, x_{T,t}^{HS}, x_{T,t}^{HB}, \{x_{N,t}^{HFS}, x_{N,t}^{HFB}\}_{F\in\mathcal{D}}} U^H(C_t^H) + e^{-\rho} E_t\left[U^H(C_{t+1}^H)\right],$$

subject to the market clearing and budget constraints

$$\begin{aligned}
&\textstyle\sum_H x_{T,t}^{HS} = \sum_H x_{T,t+1}^{HS} = 1; \quad \sum_H x_{T,t}^{HS} = \sum_H x_{T,t}^{HS} = 0,\\
&\textstyle\sum_H x_{N,t}^{HFS} = \sum_H x_{N,t+1}^{HFS} = 1; \quad \sum_H x_{N,t}^{HFS} = \sum_H x_{N,t+1}^{HFS} = 0, \qquad \forall F \in \mathcal{D},
\end{aligned}$$

$$C_{T,t}^H + \Delta_{N,t}^H P_{N,t}^H \mathbb{1}_{\{H\in\mathcal{D}\}} + S_{T,t} x_{T,t}^{HS} + B_{T,t} x_{T,t}^{HB} + \sum_{F\in\mathcal{D}} S_{N,t}^F x_{N,t}^{HFS} + \sum_{F\in\mathcal{D}} B_{N,t}^F x_{N,t}^{HFB} \le W_t^H,$$

$$C_{T,t+1}^H + \Delta_{N,t+1}^H P_{N,t+1}^H \mathbb{1}_{\{H\in\mathcal{D}\}} \le x_{T,t}^{HS}\Delta_{T,t+1} + x_{T,t}^{HB} + \sum_{F\in\mathcal{D}} x_{N,t}^{HFS}\Delta_{N,t+1}^F + \sum_{F\in\mathcal{D}} x_{N,t}^{HFB} P_{N,t+1}^F,$$

where $C^H = \{C_T^H, C_N^H\}$ denotes the standard CES consumption aggregator as in Section 2, U^H denotes the power utility function of C^H, and W_t^H denotes investor H's initial wealth. Identity operator $\mathbb{1}_{\{F\in\mathcal{D}\}}$ equals one if F is an developed country and zero otherwise, which simply reflects the fact that investors can invest in financial assets paying nontraded goods and convert these payoffs into units of traded good at the respective nontraded price P_N^F, $\forall F \in \mathcal{D}$ for their consumption purpose. In contrast, no assets paying nontraded goods of emerging markets exist, and consequently no investors, domestic or otherwise, ever need to convert these goods into the traded good and back. In other words, in the current incomplete market setting, nontraded shocks are identical to preference shocks. Furthermore, we note that by summing all countries, the above budget constraints and market clearing conditions automatically imply the resource constraints $\sum_H C_{T,t}^H = \Delta_{T,t}$, $\sum_H C_{T,t+1}^H = \Delta_{T,t+1}$ in both periods.

First order conditions corresponding to variations about optimal holding positions x_T^{HS}, x_T^{HB},

[30]With an incomplete market, the centralized optimization can also be formulated as in Pavlova and Rigobon (2008) using the convex duality technique (Cvitanic and Karatzas (1992)). However, this approach offers an exact and analytical solution only for the special case of log utility.

x_N^{HFS}, x_N^{HFB}, respectively, generate pricing equations for all available financial assets,

$$S_{T,t} = E_t\left[\frac{M^H_{T,t+1}}{M^H_{T,t}}\Delta_{T,t+1}\right]; \qquad B_{T,t} = E_t\left[\frac{M^H_{T,t+1}}{M^H_{T,t}}\right] \qquad \forall H,$$
$$S^F_{N,t} = E_t\left[\frac{M^H_{T,t+1}}{M^H_{T,t}}\Delta^F_{N,t+1}P^F_{N,t+1}\right]; \qquad B^F_{N,t} = E_t\left[\frac{M^H_{T,t+1}}{M^H_{T,t}}P^F_{N,t+1}\right] \qquad \forall F\in\mathcal{D}, \ \forall H,$$

where $M^H_{T,t} = \frac{\partial U^H}{\partial C^H_{T,t}}$ is the *country-specific* marginal utility of the *traded* consumption[31]

In the complete market setting, the marginal utilities are necessarily equalized across countries $\frac{M^H_{T,t+1}}{M^H_{T,t}} = \frac{M^F_{T,t+1}}{M^F_{T,t}}$ $\forall\{H,F\}$, which together with market clearing conditions, then establishes directly the equilibrium consumption allocations. In the incomplete market, the marginal utilities are indirectly connected to one another only through the pricing of available assets. Accordingly, the solution approach here is very different. In sequence, we first conjecture a solution for the consumption allocations, solve for the asset prices, and verify that these prices support the conjectured consumptions in equilibrium. As before, we log-linearize the above first order conditions for all countries H and all developed countries $F\in\mathcal{D}$[32]

$$\log\left(\frac{S_{T,t}}{B_{T,t}}\right) = Cov_t\left[dm^H_{T,t+1}, \delta_{T,t+1}\right]; \qquad \log\left(\frac{S^F_{N,t}}{B^F_{N,t}}\right) = Cov_t\left[dm^H_{T,t+1}, \delta^F_{N,t+1}\right]; \tag{30}$$
$$\log B_{T,t} = E_t\left[dm^H_{T,t+1}\right] + \frac{1}{2}Var_t\left[dm^H_{T,t+1}\right]; \qquad \log\left(\frac{B^F_{T,t}}{B_{T,t}}\right) = Cov_t\left[dm^H_{T,t+1}, P^F_{N,t+1}\right],$$

where dm^H denotes the log-linearized stochastic discount factor (recall from (6) that $\alpha^H \equiv \frac{1}{\gamma\omega^H_T+\epsilon\omega^H_N}$),

$$dm^H_{T,t+1} \equiv m^H_{T,t+1} - m^H_{T,t} = \log\left(\frac{M^H_{T,t+1}}{M^H_{T,t}}\right) = -(\gamma-\epsilon)\omega^H_N d\delta^H_{N,t+1} - \frac{1}{\alpha^H}dc^H_{T,t+1}. \tag{31}$$

Equilibrium: Consistent with the log-linearization approximation scheme, we look for the equilibrium consumption allocations in the following most general log-linear form,

$$dc^H_{T,t+1} \equiv \log\left(\frac{C^H_{T,t+1}}{C^H_{T,t}}\right) = g^H + a^H d\delta_{T,t+1} + \sum_F b^{HF} d\delta^F_{N,t+1} \qquad \forall H, \tag{32}$$

[31] We recall that the current numeraire is the traded good, and therefore $M^H_{T,t} = e^{-\rho t}\omega^H_T (C^H_{N,t})^{-\epsilon}\left[\omega^H_T(C^H_{T,t})^{1-\epsilon} + \omega^H_N(C^H_{N,t})^{1-\epsilon}\right]^{\frac{-\gamma+\epsilon}{1-\epsilon}}$ is the country H's pricing kernel with respect to this numeraire.

[32] Although the log-linearization technique remains useful to obtain an approximate closed-form solution, it does not address the possible multiplicity and stability of the equilibrium.

and g's, a's, b's are constant parameters to be determined, and $d\delta$'s denote the changes in log outputs, i.e., output growths ($dt = 1$)

$$d\delta_{T,t+1} \equiv \delta_{T,t+1} - \delta_{T,t} = \mu_T dt + \sigma_T dZ_T; \qquad d\delta^H_{N,t+1} \equiv \delta^H_{N,t+1} - \delta^H_{N,t} = \mu^H_N dt + \sigma^H_N dZ^H_N$$

This choice renders a log-linear SDF dm^H in the approximation and greatly simplifies the pricing of financial assets in incomplete market settings. Indeed, substituting the above conjectured consumptions and SDFs into the pricing equations and the market clearing conditions readily yields the following consumption allocations,[33] where we recall that $\alpha^I \equiv \frac{1}{\gamma\omega^I_T + \epsilon\omega^I_N} > 0$ denotes the country-specific weighted elasticity of consumption substitution.

- incomplete market: H is an emerging economy ($H \notin \mathcal{D}$)

$$c^H_{T,t} = g^H t + \frac{K\alpha^H}{\sum_I \alpha^I}\delta_{T,t} - \alpha^H \sum_{F \notin \mathcal{D}} \delta^F_{N,t} + \frac{(\gamma - \epsilon)\alpha^H}{\sum_I^K \alpha^I} \sum_{F \in \mathcal{D}} \alpha^F \omega^F_N \delta^F_{N,t}. \tag{33}$$

- incomplete market: H is a developed economy ($H \in \mathcal{D}$)

$$\begin{aligned} c^H_{T,t} &= g^H t + \frac{K\alpha^H}{\sum_I \alpha^I}\delta_{T,t} + \frac{\alpha^H \sum_{I \notin \mathcal{D}} \alpha^I}{\sum_{J \in \mathcal{D}} \alpha^J} \sum_{F \notin \mathcal{D}} \delta^F_{N,t} \\ &+ \frac{(\gamma - \epsilon)\alpha^H}{\sum_I^K \alpha^I} \sum_{F \in \mathcal{D}} \alpha^F \omega^F_N \delta^F_{N,t} - (\gamma - \epsilon)\alpha^H \omega^H_N \delta^H_{N,t}, \end{aligned} \tag{34}$$

where g^H's are country-specific parameters. These parameters help to enforce, and thus can be found from the market clearing conditions, but because they are deterministic factors, they do not enter the analysis below. To verify these equilibrium consumptions, we substitute them back into the above pricing equations to compute all available asset prices $\{S_{T,t}, B_{T,t}\}$, $\{S^F_{N,t}, B^F_{N,t}\}_{F \in \mathcal{D}}$, which finance these consumptions by the construction of the solution. This configuration is in equilibrium,[34] because, for each available asset, the associated price is identical under all investors' perspectives in the construction. Compared with the counterpart complete market setting with a single traded good, in which the consumption allocations are[35]

[33] Specifically, the pricing equations $\log(S_{T,t}/B_{T,t})$'s determine coefficients $\{a^H\}_{\forall H}$, $\log(S^F_{N,t}/B^F_{N,t})$'s determine $\{b^{HF}\}_{\forall F \in \mathcal{D}, \forall H}$, $\log B_{T,t}$'s determine $\{b^{HF}\}_{\forall F \notin \mathcal{D}, \forall H}$, and $\log(B^F_{T,t}/B_{T,t})$'s determine the nontraded prices of developed countries $\{P^F_{N,t+1}\}_{\forall F \in \mathcal{D}}$.

[34] Although this is not necessarily the unique equilibrium.

[35] This is a straightforward generalization of (5) (in the basic model) to the setting where countries have heterogeneous consumption tastes (but countries' sizes are homogeneous). In the current case, the log-linearization of FOC

- complete market: $c^H_{T,t} = g^H t + \frac{K\alpha^H}{\sum_I^K \alpha^I}\delta_{T,t} + \frac{(\gamma-\epsilon)\alpha^H}{\sum_I^K \alpha^I}\sum_F \alpha^F \omega^F_N \delta^F_{N,t} - (\gamma-\epsilon)\alpha^H \omega^H_N \delta^H_{N,t}$,

the incomplete market allocations (33), (34) are markedly different in several aspects.[36] First, the traded shock impacts stay the same in both market configurations. This is because even when the market is incomplete, the equity and bond on the traded output δ_T are available to all investors, who then are able to mitigate these shocks as optimally as possible by trading these financial assets. When combined with the force of cross-country diversification in the traded sector, this result implies that country-specific traded output risks remain relatively less material to countries' risk free rates, compared with the nontraded output risk.

Second, the nontraded output shocks (in δ^F_N) of a developed country $F \in \mathcal{D}$ affect the traded consumption c^H_T of *all* other countries H similarly, regardless of the market's completeness. Because investors can trade the financial assets contingent on these nontraded shocks, their associated risk can be shared effectively. In particular, all else being equal, a surge in developed country F's nontraded output prompts F to trim its traded consumption and boosts other countries' traded consumption by forces of trades and market clearings. Similar to the complete market settings, under a mild degree of home biases, a country's own nontraded shocks matter quantitatively more to a developed country's consumption allocation than do the nontraded shocks of their *developed* trade partners.

Third, the nontraded output shocks (in δ^F_N) of an emerging country $F \notin \mathcal{D}$ are uniformly compounded in the consumptions c^H_T of all developed countries $H \in \mathcal{D}$.[37] This feature is intuitive. In the absence of financial assets in emerging markets, these shocks cannot be properly hedged. The developed investors instead opt to simply pool their consumptions uniformly to cope with the associated risk. Risk sharing is still feasible, albeit imperfect, because it is evident from the equilibrium allocation that a surge in the nontraded output from an *emerging* economy boosts traded consumptions of all *developed* economies. The coefficient characterizing this relationship, $\frac{\sum_{I\notin\mathcal{D}}\alpha^I}{\sum_{J\in\mathcal{D}}\alpha^J}$, increases (decreases) with the number of emerging (developed) economies. That is, the significance of the unhedged risk on consumption allocations is larger when the financial market is less complete in this pooling equilibrium.

Fourth, the incomplete market has a strong and surprising impact on risk sharing between

implies $m_T = -\rho t + \omega^H_T - (\gamma-\epsilon)\omega^H_N \delta^H_N + \frac{1}{\alpha^I} c^H_T$. Combining this FOC with the (log-linearized) market clearing condition (26) for traded good yields this log consumption c^H_T in complete market. See further details in appendix D.

[36] In light of the possible existence of other incomplete market settings and multiple equilibria, our discussion here pertains to the specific incomplete market setup and the associated equilibrium presented earlier in this section.

[37] That is, $\frac{\partial c^H_T}{\partial \delta^F_N}$ is same for all $F \notin \mathcal{D}$, $H \in \mathcal{D}$.

two emerging economies. Possessing no financial assets directly tied to the nontraded output shocks of their own or those of their emerging trade partners, the emerging economies also pool their traded consumption in equilibrium to uniformly share nontraded risk. Emerging country H's traded consumption c_T^H (33) decreases with not only its own nontraded good endowment δ_N^H but also with other emerging countries' nontraded output δ_N^F. The latter behavior, which is inverse to what happens in a perfect financial market, signals that the risk sharing is most severely hampered between emerging trade partners. This is indeed the group of countries whose nontraded output risk is the least hedgeable because of the incompleteness of the market.

The incomplete market setting, as formulated in this section and pertaining to the pooling equilibrium, does not qualitatively change the risk sharing behaviors, and thus prices, among developed economies. Any sizable effects stemming from market incompleteness instead arise in the group of emerging countries whose financial markets are the least developed in the setting.